A Humean Theory of Choice of which Rationality May Be One Consequence
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Although David Hume’s place within the history of economic thought remains undisputable, his importance regarding the birth of what was to become the theory of choice has seldom been emphasized. Concerning recent decades, J.A. Schumpeter bears some responsibility in this situation: “[Hume’s] economics”, he said, “has nothing whatever to do with either his psychology or his philosophy” (Schumpeter 1954: 474n.). However, for the modern reader who considers economic theory of choice as a special case of a more general theory of action, David Hume’s discussion of the determinants of action in the *Treatise of Human Nature* (1739-1740), in the two *Enquiries*, concerning respectively *Human Understanding* (1748) and the *Principles of Morals* (1751), as well as in the *Dissertation on the Passions* (1757), deserves a more careful attention.

At first sight, Hume’s position on this matter is well known, and has given rise to an abundant literature. As numerous commentators noted it, Hume sharply distinguished ‘passion’ and ‘reason’, respectively dedicated to the determination of action, and to the separation between truth and falsehood. On the one hand, this means that reason cannot be opposed to passion in the determination of action; and, on the other hand, that reason is subordinated to passion. As Hume stated it in an often quoted passage:
“We speak not strictly and philosophically when we talk of the combat of passion and of reason. Reason is, and ought only to be the slave of the passions, and can never pretend to any other office than to serve and obey them” (THN II: 415).

Of course, both the ability and the autonomy of such a slave might be discussed. From this point of view, it should be recalled that Hume distinguished two different domains in which reason is operating: ‘relations of ideas’, and ‘matters of fact, or existence’. The first deals with formal reasoning, illustrated by mathematics, and gives birth to certain knowledge, whereas the second cannot reach such a certainty. According to Hume, “the first species of reasoning” alone cannot be considered as the cause of any action: “As it’s proper province is the world of ideas, and as the will always places us in that of realities, demonstration and volition seem, upon that account, to be totally remov’d, from each other” (THN II: 413). At the very most, Hume grants that “[a]bstract or demonstrative reasoning” influences our actions “only as it directs our judgement concerning causes and effects” (Ibid: 414). The content of this direction of judgement is made clear by the examples provided in the same passage: they concern the arithmetic involved in the laws of mechanics, or in the calculation of the debts of a merchant towards a possible creditor. The second kind of reasoning, which points at what Hume calls ‘proof’ – as opposed to ‘demonstration’ – and ‘probability’, is mentioned through the relation between cause and effect. Hume does not give any immediate illustration, but he explains that our judgement leads us to an extension of the number of objects that our passions affectively invest:

“‘Tis obvious, that when we have the prospect of pain or pleasure from any object, we feel a consequent emotion of aversion or propensity, and are carry’d to avoid or embrace what will give us this uneasiness or satisfaction. ‘Tis also obvious, that this emotion rests not here, but making us cast our view on every side, comprehends whatever objects are connected with its original one by the relation of cause and effect” (THN II: 414).

The consequence is plain: though reason is, in some manner, involved in the determination of action, Hume does not seem to give any evidence which would favour the kind of involvement of reason that we identify in modern theories of rational choice. Such is the interpretation of some modern commentators, like R. Sugden (1991) or E. Picavet (1996).

This paper aims at challenging this interpretation. After recalling that the question of rationality in the theory of decision under certainty has led to an investigation concerning the completeness and the transitivity of the preference relation, and the compatibility between preferences and choice, as special cases of what is usually considered as ‘rational choice’ (§1.1.), some relevant features of Hume’s theory of passions, regarding decision theory, are put to the fore. They prevent us from considering the set of choice or the domain of choice as pre-existent to the preference relation, and prompt us to recognize the analytical link between desire and will in Hume’s theory of passion, and preference and choice in decision theory (§1.2.). The completeness of the preference relation thus appears as a consequence of the priority, that Hume establishes, of the passions over the objects which they point out, whereas
compatibility between preferences and choice corresponds to Hume’s conception of the relationship between desire and will (§1.3.). Hume’s emphasis on an endogenous dynamics of passions based on a “double relation” – relation of ideas, and relation of impressions – gives rise to some formal statements concerning the revision of the preferences and of the set and domain of choice (§2.1.). Although quite general, these statements allow the construction of a Humean decision algorithm which, in particular, displays a choice function rationalizable at each of its steps (§2.2). Grounded on some stabilization properties suggested by Hume, the last feature of rationality, transitivity, then appears not as a pre-established characteristic of the preference relation, but as an outcome of the decision process (§2.3.).

1. THE PASSIONATE EXISTENCE OF OBJECTS

1.1. Prelude: In which way could a choice be rational?

The question of knowing on which grounds does ‘rationality’ rest, according to modern rational choice theory, has not a clear-cut answer. However, we are much indebted to K. Arrow who first invited his colleagues, in 1951, to give up the recourse to the techniques of differential calculus in favour of an approach till then developed by logicians, and to explain choice as the outcome of a preference relation R defined over a set of reference of choice X. Since that time, and for most economists – as Arrow (1967: 5) again, noted it – it is usually admitted that R is a preorder, which means that it is:

- reflexive (\( \forall x \in X, x \mathcal{R} x \)),
- complete (\( \forall x, y \in X, x \mathcal{R} y \) or \( y \mathcal{R} x \)), and
- transitive (\( \forall x, y, z \in X, x \mathcal{R} y \) and \( y \mathcal{R} z \Rightarrow x \mathcal{R} z \)).

The axioms of completeness and transitivity both provide possible (and not exclusive) interpretations to the rationality of choice. This latter hence refers:

1. to some kind of cognitive ability, in the sense where the comparability between two arbitrary elements of X is independent of its dimension. Lack of completeness thus appears less as non-rationality than as a limitation to rationality, as ’bounded rationality’ – which amounts to consider, according to H. Simon’s brilliant expression, “mind as the scarce resource” (H. Simon 1978: 9);
2. or, to a necessity of the mind, which can hardly be ruled out from a normative point of view. Such a necessity seems only to require that transitivity be restricted to the asymmetric part of R, that is to strict preferences, so that it might be replaced by quasi-transitivity (\( \forall x, y, z, \in X, [(x \mathcal{R} y \text{ and } \neg y \mathcal{R} x) \text{ and } (y \mathcal{R} z \text{ and } \neg z \mathcal{R} y)] \Rightarrow x \mathcal{R} z \)) or acyclicity
\[(\forall t \in \mathbb{Z}^+, \forall x^1, x^2, \ldots, x^t \in X, [\forall \tau \in \{1, 2, \ldots, t-1\} : (x^\tau R x^{\tau-1} \text{ and } \neg x^{\tau-1} R x^\tau)] \Rightarrow \neg (x^t R x^1 \text{ and } \neg x^1 R x^t))\] – which both give room to some kind of preference indiscrimination.

However, in spite of the problems raised by quasi-transitivity or acyclicity – such as the impossibility to represent preferences by a single-valued numerical function like a utility function – the resulting ‘order’ appears to meet current requirements concerning the rationality of preferences.

Presumably less familiar, an other line of interpretation of the rationality of choice might be derived from the pioneering work of P. Samuelson on revealed preferences (Samuelson 1938). As is well known, this latter carefully distinguished between ‘preferences’ and ‘choice’. This distinction is essential to introduce M. Richter (1971)’s conception of rational choice as:

(3) the compatibility between preferences and choices, in the sense where an agent, alternatively facing various “contexts of choice” (or “opportunity set”, or “budget”, according to alternative terminologies) denoted S, would always choose what he prefers. More formally, this means that, given the set of choice X, and a set of subsets of X (the domain of choice) denoted F, a choice function C(.) is rational if there exists a binary relation R, such that \(\forall S \in F\), the sets of optimal elements of R being \(G(S, R) = \{x \in S : x R y, \forall y \in S\}\), \(C(S) = G(S, R)\). On the one hand, such a definition calls to mind the Aristotelian conception of akrasia, according to which irrationality stems from a discrepancy between what an agent considers as his best choice and what he actually chooses. On the other hand, it opens the path to a more qualified perception of the domain of choice. Still to day, for most economists, it is obvious that F is abstract, that is, if \(P(X)\) is the set of all subsets of X, \(F = P(X) \setminus \emptyset\). Nonetheless, F could as well be selective, that is, \(F \subset P(X) \setminus \emptyset\). Now, this possibility is far from being without consequences. For instance, whereas it could be shown that when C(.) respects the Weak Axiom of Revealed Preference (WARP)\(^2\), if F is abstract, rationality in the sense (2) is equivalent to rationality in the sense (3), such a strong result disappears in the more general case of a selective domain of choice: we might choose what we prefer (3); this does not mean that our preferences are transitive (2).

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1 As a result of a lack of discrimination, R. Luce (1956), for instance, constructed the concept of ‘semi-order’, in which strict preferences are transitive but indifference is not necessarily transitive – which means that R is quasi-transitive.

2 See infra, note 2, p. 19.
Leaving aside the question of choice under uncertainty, the question of rationality in the theory of decision hence leads to an investigation concerning (1) **completeness**, (2) **transitivity** and, (3) **compatibility** as special cases of what is usually considered as ‘rational choice’. Now, David Hume’s work, although in a non-technical fashion, supplies possible treatments of these three alternative conceptions of rationality.

1.2. From desire and will to preferences and choice

Devoted to the study of ‘passions’, book II of Hume’s *Treatise* is based, like his later published *Dissertation*, on a distinction between direct passions (joy and grief, hope and fear, desire and aversion, volition) which “arise immediately from good or evil, from pain or pleasure” (*THN II*: 276), and indirect passions (typically, pride and humility, love and hatred) for which, although they “proceed from the same principles” (*Ibid.*), pleasure and pain appear as their outcomes. As Hume explained it, the indirect passion is “plac’d betwixt two ideas, of which the one produces it, and the other is produc’d by it. The first idea, therefore, represents the *cause*, the second the *object* of the passion” (*THN II*: 278). Quite naturally in the *Dissertation*, Hume first discusses the direct passions, and then, the more complicated structure of indirect passions. Now, the *Treatise* displays an apparently more disputable order of presentation, in which the indirect passions are first examined. As several scholars pointed it, this should be related to Hume’s intention to provide a criticism of the substantial self: Hume’s conception of pride\(^1\) leads to argue that this latter is not a consequence of some pre-established disposition of the self but, on the contrary, that the self is constituted as an *object* of pride\(^2\).

On the one hand, this does not seem to challenge seriously our way of thinking, at least as economists. It suggests that for Hume, the self comes to existence only through his/her perceptions, and that when we speak of the ‘agent’ of a choice, we should keep in mind that this quality – being an agent – is a result of a passionate effect. But on the other hand, such an analysis has more thought-provoking implications. It is well-known that, dealing with the substantial self, Hume extended Berkeley’s philosophically subversive position, according to which exterior objects have no other existence than in the perception

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1 In D. Davidson’s words, Hume has a *propositional* theory of pride, which concerns the fact of being proud of something, instead of being a proud person (Davidson 1976). See also P. Árdal 1966: 16 sqq and 1989.

2 Hume’s taxonomy of indirect passions is based on a double criterion: the object might be either ‘self’ *(pride and humility)* or ‘another’ *(love and hatred)*; the associate sensation might be either ‘agreeable’ *(pride and love)* or ‘disagreeable’ *(humility and hatred)*. This taxonomy is conceived in such a way that all other indirect passions (ambition, vanity, envy, pity, malice, generosity…) appear as variants of the first ones.
which we have of them: their existence is this perception. So that the objects possibly submitted to our choice are not objects that would exist independently of our perceptions and that our reason would point to us, but the only objects the existence of which we acknowledge through a passionate investment:

“Where the objects themselves do not affect us, their connexion can never give them any influence; and 'tis plain, that as reason is nothing but the discovery of this connexion, it cannot be by its means that the objects are able to affect us” (THN II: 414).

Even mathematics cannot be considered as an independent activity of the mind, in which reason would be coping with pre-existent entities: like in philosophy (THN II: 448-54), only the passion of curiosity, or the love of truth, gives rise to such curious things like angles, lines or proportions, which would have never come to birth without it.

As a result, from a Humean perspective, the question of choice should be asked in a quite different way. We are used to consider that the set of choice X, the domain of choice F ⊆ P(X) \ ∅, the contexts of choice S ∈ F, all exist independently of the agent, and that his/her emotional or passionate state expresses itself by means of a relation of preferences R or a choice function C(S). As noticed above, such a representation leaves room to a possible interpretation of rationality as cognitive ability, ensuring the comparability between any arbitrary pair x and y, either of X or – if the existence of alternative contexts S belonging to the domain of choice F is explicitly taken into account – of S ∈ F. On the contrary, Hume’s theory of passions prevents us from considering X or F as given: they come to the world for us through a passionate investment. From an economist’s point of view, this would have remained negligible, had not this passionate investment been also involved in what is to day considered as preferences and choice.

Within Hume’s theory of passions, the decision process could be analysed as the ultimate step, leading to action, of a wider process describing the transformations of our passionate state. In this ultimate step, the above-mentioned direct passions play a crucial part. Indeed, this system of direct passions runs like a funnel, which starts with joy and grief, continues with desire and aversion, and reaches completion with volition or will which

1 More precisely, Hume argues that “[t]he only existences, of which we are certain, are perceptions, which being immediately present to us by consciousness, command our strongest assent” (THN I: 212).

2 Although the perspective adopted here is quite different, such a way to consider that preferences do not exist prior to the choice situation but are constructed within the decision process, should not be surprising to those who are familiar to the works of psychologists on preferences and choice. See, for example, A. Tversky 1996.

3 This enumeration leaves aside the ‘mixed’ passions of hope and fear, which appear as immediately linked to the question of choice in uncertainty (see A. Lapidus 2000: 51 sqq).
immediately precedes action: joy and grief are the most straightforward ways of feeling respectively pleasure and pain; desire and aversion both represent this state of the mind which constitutes comparable objects; and finally, the will, as Hume’s own words, is “nothing but the internal impression we feel, and are conscious of, when we knowingly give rise to any new motion of our body, or new perception of our mind” (THN II: 399). In this decision process, desire and aversion on the one hand, volition on the other, respectively refer to preferences and choice. The relation between volition and choice seems obvious: if it is not transformed by another passionate movement, volition consists, like a standard choice function, in selecting an alternative which is to be followed by the corresponding action.

Meanwhile, the similarity between desires and preferences, although currently admitted, requires some more explanations, since preferences are represented by binary relations between objects, whereas desires only seem to point at such or such object. It should then be recalled that Hume also provides a basis for the comparison between desires. For instance, after having introduced the role of instincts in the birth of desire, he mentions “the general appetite to good, and aversion to evil, consider’d merely as such” (THN II: 417). The same idea is expressed some pages farther, when he explains that desire “arises from good, consider’d simply” (THN II: 439). This should convince the reader of the Treatise that, in spite of the fact that desires relate to heterogeneous objects, they are made homogeneous since they concern “simply” or “merely” good and evil – that is, according to Hume, pleasure and pain.

1 “[G]ood and evil, or in other words, pain and pleasure […]” (THN II: 439).

2 See, for example, THN I: 119. The force of an idea of an impression is what Hume calls the “belief” in this impression.
aversions may be viewed as a simple expression of an underlying binary relation of preference, so that “x is more desired than y” means “x is preferred to y”1.

However, this is not completely satisfying. It is clear, either from the Treatise or from the Dissertation, that when Hume deals with desires, he considers them as comparable desires; but this does not explain how we pass from a direct desire for x or for y to a comparative desire between x and y: even if it is possible for me to compare my desire for x and my desire for y, how is it that I establish a link between them, so that the differences between my desires make sense for me? Hume’s well-known answer favours the “natural relations” of the mind, this binary relation2 which leads the imagination both from one idea to another, and from one impression to another. More particularly studied in the first book of the Treatise, these natural relations (resemblance, contiguity, and causality; THN I: 10-3) express the most immediate functioning of our imagination3, the “gentle force” (THN I: 10), the “gentle and insensible movement” (EHU: 50), which constructs for our sake comparable desires for different objects.

1.3. Rationality as completeness, and rationality as compatibility

Now, such a perspective determines our way of considering rationality, at least in two of the three senses pointed out in the previous section: rationality as completeness and rationality as compatibility.

Rationality as completeness results from the fact that desires or preferences are not defined on a pre-existent context of choice, but that they stem, with the objects that they concern, from the same passionate configuration. It is then obvious that exterior objects which are not invested by this passionate configuration, do not belong to the context of choice: they do not arouse either desire, or aversion. For the agent at this moment, his/her reason cannot take them into consideration: they simply do not exist. The passionate configuration, which

1 From another point of view, it should be noted that since desires refer, as ‘impressions of reflection’, to either pain or pleasure, they supply something more than preferences. They assume the existence of some kind of zero-value, which separates aversion and desire, and which is lacking in the usual conception of preferences. This makes significant the fact that some combinations of objects might be at the border between desire and aversion. In spite of its importance for the rest of Hume’s construction, this dimension of desire and aversion will be neglected hereafter.

2 D. Garrett 1997: 250 n. 10 nonetheless mentions the possibility to generalize Hume’s approach to relations of an order greater than two.

3 In the enthusiasm of the Abstract, Hume concludes that the natural relations “are really, to us, the cement of the universe, and all the operations of the mind must, in a great measure, depend on them” (ATHN: 662).
leads to desire and aversion, hence produces a contextual preference relation $R_S$ defined over a context of choice $S$, such that:

$$\forall x \in S, \exists y \in S : x R_S y \text{ or } y R_S x \quad [1.1].$$

The relations between the objects belonging to the context of choice $S$ that our passions invest, are ruled by the so-called “natural relations” – resemblance, contiguity, and causality – mentioned above. The important point, here, is that these natural relations, which only associate ideas, should be distinguished from what Hume names “philosophical relations”\(^1\), which compare ideas, and depend on the particular circumstance by which we deliberately compare two ideas “without a connecting principle” (THN I: 14). Here, nothing is deliberate. Our mind passes, according to its own necessity, from one object to another, and the objects linked by such a connection are those which constitute the context of choice. Stronger than the simple contextualization expressed in [1.1], this connection may be written:

$$\forall x, y \in S, \exists t_1, \ldots, t_i, t_{i+1}, \ldots t_n \in S : (x R_S t_1 \text{ or } t_1 R_S x), \ldots (t_i R_S t_{i+1} \text{ or } t_{i+1} R_S t_i), \ldots (y R_S t_n \text{ or } t_n R_S y) \quad [1.2].$$

Moreover, such connections within our imagination display a quite interesting property, which Hume emphasizes as follows:

“That we may understand the full extent of these relations, we must consider, that two objects are connected together in the imagination, not only when the one is immediately resembling, contiguous to, or the cause of the other, but also when there is interposed betwixt them a third object, which bears to both of them any of these relations” (THN I: 11).

In other words, the ‘extent’ of natural relations means that,

$$\forall x, y, z \in S : x \neq y \neq z, \quad ((x R_S y \text{ or } y R_S x) \text{ and } (y R_S z \text{ or } z R_S y)) \Rightarrow (x R_S z \text{ or } z R_S x) \quad [1.3].$$

Allowing reflexivity, that is:

$$\forall x \in S, x R_S x \quad [1.4],$$

the completeness of the contextual preferences $R_S$ over $S$ clearly results from the connection [1.2] that natural relations establish between the ideas of the objects that we desire, and from the extent [1.3] of these relations.

It might be objected to this presentation that, since $R_S$ is a binary relation over $S$, it is difficult to admit that $S$ is not given strictly before it. This objection can however be bypassed. We can define a *contextual set of preferences* $Q^S$ as a subset of the Cartesian product

\[^1\text{Nonetheless, resemblance, contiguity and causality are considered as natural relations when they associate (THN I: 10-3; 92-3), and as philosophical relations when they compare (Ibid.: 69-78).}\]
E \times E$, where $E$ stands for the universal set of objects. $Q^S$ is therefore a binary relation over $E$. From $Q^S$, it is always possible to determine its support, the context of choice $S = \text{Supp} (Q^S) = \{x: (x,y) \text{ or } (y,x) \in Q^S\}$, and contextual preferences $R_S = \{(x,y) \in Q^S : x, y \in S\}$. It should be stressed that the contextual preference $R_S$ is a binary relation over $S$, whereas $Q^S$ is a binary relation over $E$. So that $R_S$ can alternatively be understood as the restriction of $Q^S$ over $S$. The priority granted to $R_S$ in a Humean perspective thus only means that we are given the set of preferences $Q^S$, and that although the context of choice $S$ is not given, it can be derived from $Q^S$ as its support. The usual approach of preferences would, on the contrary, favour a possibly larger context of choice, let us say $T \supseteq S$, over which $R_S$ is defined. In such a case, contrary to the Humean perspective, the conditions of contextualization [1.1] and of connection [1.2] above would not be fulfilled, and consequently, the completeness of $R_S$ over $T$ could not have been established.

A consequence of this way to conceive both preferences and the objects that they concern must be underlined. A standard objection to the application of instrumental rationality through an optimisation process should, indeed, be pushed aside. If such an optimisation represents a cost for the agent, knowing if it is advisable to optimise entails, again, another optimisation, and the infinite regression that follows may hinder the process of optimisation itself. Now, this eventuality is clearly excluded by the perspective adopted by Hume. The expression of preferences does not correspond to a pain-generating operation, which would take place only after the recognition of the objects which these preferences concern: the identification of objects and the structure of desires into which they fit is an effect of the same passionate investment, and depends on the same movements of the imagination. It is therefore only in a metaphoric way that the Humean procedure of choice can be understood as a “calculation” involved in optimisation. It does not lead to additional pains which would pervert the arbitration between initial pleasures and pains: it is a natural manifestation of the mind, which contributes to the constitution of desire and aversion, but which is, as such, the object neither of desire, nor of aversion. So that when we neglect the objection concerning infinite regression, we are not necessarily Humean, but Hume’s approach may justify our theoretical option.

Rationality as compatibility between preferences and choice can be tackled through the place that Hume grants to the will, in relation to other direct passions, especially desire and aversion. We previously argued that for Hume, the will is to be considered among direct passions. Indeed, there is some textual evidence which favours this interpretation: the will is included in the enumeration of the section of the Treatise dedicated to the direct passions (THN II: 438), as well as in that of the Dissertation (DP: 139). However, this requires some qualification: the will is not included in the list which appears at the beginning both of the first part of Book II of the Treatise (THN II: 277), and of its third part concerning ‘the will
and direct passions’ (THN II: 399). More precisely, in this last passage, Hume writes that it is “not comprehended among the passions” (Ibid.).

This seemingly contradictory presentation makes apparent the narrow path that Hume borrows. On the one hand, if the will is excluded from the list of passions, it might be understood as a faculty, either autonomous or governed by reason, hence liable to determine action and to take precedence over passions. Since Hume is especially interested in presenting action as the result of a passionate necessity, contrasting with the doctrine of free will, it becomes clear that the will has to be counted among the passions. But on the other hand, considering that the will is itself a passion might allow to understand it as another determinant of action, which would have its own informational content. Now, the will says nothing else than the desire already said – except that it is to be followed by action when it is not altered by a transformation of passions: it “exerts itself”, Hume writes, “when either the good or the absence of the evil may be attain’d by any action of the mind or body” (THN II: 439). In other words, it extends to action the prescriptions of desire and aversion. In spite of its importance, the will hence appears to remain exterior to passions, to which it would add nothing.

One might discuss at length the question of knowing on which side of the border, between passions and actions, volition is anchored. But, if one accepts to consider that choice refers to the will, just like preferences refer to desire, it becomes possible to observe that Hume supports strongly the idea that when we are led to choose, we do not choose anything else than what we prefer. Stated more formally, this means that, if we are in a passionate state characterized by a set of preferences \(Q^S\), the elements of the assumed non-empty \(C(S)\), chosen in the context of choice \(S\) (support of \(Q^S\)), are such that \(C(S) = \{x \in S : (x, y) \in Q^S, \forall y \in S\}\) – which meets Richter’s understanding of rational choice as compatibility between preferences and choice.

It should be emphasized that, even if one agrees with the conclusion according to which Hume’s theory of passion leads to completeness and compatibility, this does not coincides exactly with the corresponding usual conceptions of rationality. These latter are both related to the set of choice \(X\), whereas completeness and compatibility seem to express here weaker types of rationality, since they are related to a context of choice \(S\). However, it could be argued that this rationality is not as weak as it seems to be. Indeed, we can imagine successive passionate states, characterized by different \(Q^S’s\), \(R^S’s\) and \(S’s\). In each case, completeness and compatibility would prevail, but the \(S’s\) should not be viewed as non-empty subsets of a pre-existent given \(X\): just as \(S\) is derived from the set of contextual preferences \(Q^S\) as its support, \(X\) is constructed as the superset of successive \(S_i’s\). For instance, a sequence of contextual sets of preferences \(Q^{S_0}, Q^{S_1}, \ldots, Q^{S_i}\), gives rise to a parallel sequence of choice
reference sets \( X_0, X_1, \ldots X_i \), where \( X_0 = S_0, X_1 = X_0 \cup S_1 \), and \( X_i = X_{i-1} \cup S_i \), and where \( S_i \) is the context of choice which supports \( Q^{S_i} \).

2. “TILL THE WHOLE CIRCLE BE COMPLEATED”: RATIONALITY AS TRANSITIVITY

The process by which successive contexts of choice \( S \), give birth to a set of reference of choice \( X \) constitutes the way transitivity will be approached in a Humean perspective. For most economists today, completeness and compatibility would not correspond to their intuitive idea of the rationality of choice, since they are associated neither to transitivity, nor to its weaker forms, like quasi-transitivity or acyclicity. According to the few commentators interested in this question (see, for instance, R. Sugden 1991: 754, or E. Picavet 1996: 69), it is generally asserted that, although there is no direct evidence for it, Hume’s theory of action provides no grounds for such a thing as transitivity. The argument is that, since desires and preferences, volition and choice, belong to what Hume calls ‘relations of facts’, transitivity (just like its opposite) would be conceivable and, accordingly, not contrary to reason. An opposite view was recently supported by one of us (A. Lapidus 2000: 30-2), which leads to understand transitivity as a consequence from the Humean necessity of non contradiction concerning facts, more precisely from the impossibility for the effect of a cause to cause its own cause.

2.1. The “double relation” and beyond: from a dynamics of passions to a decision process

Nevertheless, another way, also leading to the conclusion that Hume’s theory of choice is rational in the sense that preferences are transitive, will be followed hereafter. It rests on Hume’s attempt to explain the dynamics of passions, which was an important feature of book II of the Treatise, as well as of the Dissertation. The key to such a dynamics is a rule of double relation – relation of ideas, and relation of impressions (THN II: 282-4; DP: 144-5) – which aims at giving the conditions for which a passion stems from another passion:

“The present theory of the passions”, Hume says, “depends entirely on the double relations of sentiments and ideas, and the mutual assistance, which these relations lend to each other” (DP: 158).

The “relation of ideas” leads from the object of the first passion to the object of the second. It follows the already mentioned “natural relations”: resemblance, contiguity, and

\[1\] The principle of the “double relation” is analysed in detail in Book II of the Treatise (THN II: 282-4), where it is mentioned repeatedly (for example, Ibid.: 438-9) before being reminded again in Book III (THN III: 574).
causality (THN I: 10-3). For instance, when discussing the case of pride associated to the organization of a feast, Hume draws on contiguity and causality in order to explain, that the “feast” – the object of joy, the initial direct passion – is related to the “self” – the object of pride, the indirect passion which follows joy – for the one who is “the master of the feast” (THN II: 290). The second element of the double relation is the “relation of impressions”, based on the natural relation of resemblance\(^1\). In the previous example, the pleasure which is at the origin of the direct passion of joy is associated to the one that arouses the indirect passion of pride: the pleasure which I feel in participating in a feast meets the one of being the organizer of such a successful event.

But this is not a one shot process. The second passion does not cancel the first one: by means of the double relation, it rather amplifies and transforms it, and modifies the resulting desires and volition. Again, an example of the Treatise summarizes this dynamics of passions:

> “[A] suit of fine cloaths produces pleasure from their beauty; and this pleasure produces the direct passions, or the impressions of volition and desire. Again, when these cloaths are consider’d as belonging to ourself, the double relation conveys to us the sentiment of pride, which is an indirect passion; and the pleasure, which attends that passion, returns back to the direct affections, and gives new force to our desire or volition, joy or hope” (THN II: 439).

The succession of desires, which appear as landmarks in the transformation of passions, deserves special attention. They can be represented by a sequence of contextual sets of preferences \(Q^{S_0}, Q^{S_1}, \ldots, Q^{S_i}, \ldots\) in which the underlying contextual preferences \(R^{S_0}, R^{S_1}, \ldots R^{S_i}, \ldots\) are (as a result from [1.2] and [1.3] above) complete over \(S_0, S_1, \ldots S_i, \ldots\). Stressing the endogenous, rather than exogenous, origin of this dynamics, the embedding of the relation of ideas and of the relation of impression within Hume’s double relation aims at explaining the embedment of the movements of the imagination and of the movements of the emotions involved in the sequence \((R^{S_0}, S_0), (R^{S_1}, S_1), \ldots (R^{S_i}, S_i), \ldots\) Moreover, taking into account that, as already noted, the set of reference of choice \(X\) is constructed as the union of the contexts of choice \(S\), the way back to the initial emotions, which, in Hume’s words, “gives new force to our desire”, suggests that the emergence of a new context of choice \(S_i\) is followed by an extension of the set of choice:

\[
X_i = X_{i-1} \cup S_i \tag{2.1},
\]

over which general preferences are reformulated, leading to \(R^{X_i}\).

In the same way, the domain of choice \(F_i\) is extended in order to include both \(S_i\) and \(X_i\):

\[
F_i = F_{i-1} \cup \{S_i\} \cup \{X_i\} \tag{2.2}.
\]

---

\(^1\) Hume notes that “there is an attraction or association among impressions, as well as among ideas; tho’ with this remarkable difference, that ideas are associated by resemblance, contiguity, and causation; and impressions only by resemblance” (THN II: 283).
From a more conventional point of view, at least the extension of the set of choice [2.1] could be interpreted as an information gathering property of the choice process.

However, [2.1] and [2.2] hush up the precise way preferences are updated. But the point is that they \textit{are} updated. Discussing our love life or our intertemporal preferences (see A. Lapidus 2000), Hume tries to explain how a transformation concerning our passions\footnote{In the case of intertemporal preferences, Hume’s argument concerns the increase of the preference for the present, which corresponds to the transformation of a ‘calm’ passion into a ‘violent’ one, as a result of an enhanced efficiency of the natural relation of contiguity (see A. Lapidus 2000: 45-9).} entails that “our general resolutions are frequently confounded” (\textit{EPM}: 239). This means that the new contextual preferences $R_{Si}$ prevail over the previous general preferences $R_{Xi-1}$ and give birth to new general preferences $R_{Xi}$. It seems difficult to find more precise statements in Hume’s works about the way the ‘general resolutions’, now expressed in $R_{Xi}$, get some precedence. Nonetheless, it should concern i) the revision of contextual preferences submitted to the influence of general preferences [2.3], and, ii) the exclusion [2.4] or inclusion [2.5] of contexts of choice in the domain of choice, as a result of the compatibility of general and contextual preferences over each context:

- The minimal and most general consequences of the revision of our preferences $R_{Xi}$ is that if $R_{Si}$ and $R_{Xi}$ do not select the same non-empty subset of optimal elements over $S_i$, $R_{Si}$ should be replaced by $R_{Xi/Si}$, the restriction of $R_{Xi}$ over $S_i$. That is:

  \[ G(S_i, R_{Si}) \neq G(S_i, R_{Xi/Si}) \Rightarrow R_{Si} \leftarrow R_{Xi/Si} \]  

- On the other hand, if $R_{Xi}$ is not compatible with some $R_S$ over $S \in F_i$, in the sense that the set of optimal elements over $S$ using $R_S$ is different from the set of optimal elements over $X_i$ using $R_{Xi}$ which belong to $S$, then previous “general resolutions” expressed in this $S$ are given up, so that it is excluded from the domain of choice $F_i$:

  \[ \forall S \in F_i, G(S, R_S) \neq G(X_i, R_{Xi}) \cap S \neq \emptyset \Rightarrow F_i \leftarrow F_i \setminus \{S\} \]  

- But, if general preferences $R_{Xi/S}$ restricted to $S$ are compatible over a context $S$ previously rejected from the domain of choice (that is, belonging to a set $F_{i-1}$), with any general preferences $R_{Xi/S'}$ restricted to $S'$ which belongs to the domain of choice (that is, belonging to $F_i$), with $S \subset S'$ then $S$ should be reintegrated to it and $R_S$ should be updated:

  \[ \forall S \in F_{i-1} \setminus (F_{i-1} \cap F_i), \forall S' \in F_i, S \subset S', G(S, R_{Xi/S}) = G(S', R_{Xi/S'}) \cap S \neq \emptyset \Rightarrow F_i \leftarrow F_i \cup \{S\} \text{ and } R_S \leftarrow R_{Xi/S} \]  

\[ 2.2. \text{ A Humean algorithm of decision} \]

Focussing on the steps of the dynamics of passions when desires emerge from a passionate configuration, the algorithm of decision described hereafter is built on the above
relations [2.1] to [2.5], which constitute the significant ingredients of the Humean mental process which leads to choice through a transformation of passions. This algorithm (D-PROC) includes two parameterised procedures, respectively called EXTEND and UPDATE.

Let us stress out that through D-PROC, the agent gets a new contextual preference set $Q_{Si}$ and determines $S_i$ the associated context of choice. D-PROC first uses EXTEND and gives to it as input the current context of choice $S_i$. Likewise, EXTEND calls UPDATE and gives to it as inputs $S_i$ (the current context of choice), $X_{i-1}$ (the set of reference of choice produced by the step i-1), $F_i$ (the domain of choice) and $F_{i-1}$ (the domain of rejected contexts of choice at step i-1). When D-PROC is completed, it presents as outputs the set and the domain of choice, as well as general and contextual preferences.

**D-PROC**

\[
\begin{align*}
&i \leftarrow 0; \\
&X_i \leftarrow \emptyset; \\
&X_{i-1} \leftarrow \emptyset; \\
&F_{i-1} \leftarrow \emptyset; \\
&F_{i-1}^r \leftarrow \emptyset; \\
&\text{NewPref} \leftarrow \text{true}; \\
&/* \text{ Initialization } */ \\
&\text{While (NewPref } \not\leftrightarrow \text{ false) do;} \\
&\quad \text{Write the following message : Submit (}Q_{Si}\text{);} \\
&\quad /* \text{ The agent has a new contextual set of preference } Q_{Si} */ \\
&\quad \text{Read (}Q_{Si}\text{);} \\
&\quad S_i = \{x : (x, y) \text{ or } (y, x) \in Q_{Si}\}; \\
&\quad /* \text{ The new context of choice is determined as the support of } Q_{Si} */ \\
&\quad R_{Si} = \{(x, y) \in Q_{Si} : x, y \in S_i\}; \\
&\quad /* \text{ The new contextual preference is determined as the restriction of } Q_{Si} \text{ over } S_i */ \\
&\quad \text{If no } (Q_{Si}) \text{ is read then NewPref } \leftarrow \text{ false ;} \\
&\quad \text{If (NewPref } \not\leftrightarrow \text{ false) then do;} \\
\end{align*}
\]
EXTEND \[S_i\] ;
/* The EXTEND procedure starts here */
End ;
/* of If */
End ;
/* of While */

Write as outputs \(X_{i-1}, F_{i-1}, R_S\) for all \(S \in F_{i-1}\);

END ;
/* of D-PROC */

EXTEND \[S_i\] ;
\[X_i = X_{i-1} \cup S_i;\]
/* The set of choice is extended as noted above in [2.1] */
\[R_{Xi};\]
/*... and a complete general relation of preference is determined over \(X_i\) */
If \(i = 0\) then \(F_i = F_{i-1} \cup \{S_i\}\) else if \(i \geq 1\) then \(F_i = F_{i-1} \cup \{S_i\} \cup \{X_i\}\);
/* See above the extension of the domain of choice [2.2]. This instruction is broken in two parts in order to avoid having \(S_0\) twice */

UPDATE \[S_i, X_{i-1}, F_{i-1}\];
/*The UPDATE procedure starts here */
i \leftarrow i + 1;
END;
/* of EXTEND */

UPDATE \[S_i, X_{i-1}, F_{i-1}\];
\[R_{Xi/X_{i-1}};\]
/*The restriction to \(X_{i-1}\) of the agent’s preferences over \(X_i\) */
\[R_{Xi/Si};\]
/* The restriction to \(S_i\) of the agent’s preferences over \(X_i\) */

If \(G(S_i, R_{Si}) \nRightarrow G(S_i, R_{Xi/Si})\) then do;
\[R_{Si} \leftarrow R_{Xi/Si};\]
/*Updating of contextual preferences over \(S_i\) if they are not compatible with general preferences over \(X_i\). See [2.3] above */
End;
/*of If */

For any \(S \in F_{i-1}\) do;
Reint \leftarrow false;
For any \(S' \in F_i\) with \(S \subseteq S'\) and \(G(S', R_{Xi/S'}) \cap S \neq \emptyset\) do;
If \(G(S, R_{Xi/S}) = G(S', R_{Xi/S'}) \cap S\) then Reint \leftarrow true;
Else Reint \leftarrow false;
End;
/*of For any */
If Reint \leftarrow true then do;
The algorithm D-PROC allows to find again the previous results concerning the rationality of Hume’s decision process. It is by construction that it leads to rationality in the sense of completeness at each of its steps. The rationality in the sense of compatibility between preferences and choice, though less trivial, is easy to establish as a property of the algorithm. Proposition 1 below, shows that even if the agent has a preference relation over each context S, using the preference updating process displayed in D-PROC leads to a rational choice (as compatibility between general preferences and choice) at any step n. Therefore, D-PROC can be seen as a procedure for selecting a rationalizing profile, which includes general preferences RXn over Xn, so that the restrictions on S of these latter are rationally equivalent to each contextual relation RS belonging to this profile, in the sense that their optimum elements are identical.

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1 A ‘rationalizing profile’ over F is a profile \( \Pi = (\ldots, R_S, \ldots) \) of binary relations over \( S \in F \), such that \( \forall S \in F, C(S) = \{x \in S : x R_S y, \forall y \in S\} \). See M. Diaye 2001.
PROPOSITION 1: D-PROC, at any step n, leads to a rational choice function C(.) defined from $F_n$ to $P(X_n)$.

Proof. At the end of step n, we obtain a set of choice $X_n$, a domain of choice $F_n$, and contextual preferences $R_S$ defined over contexts of choice $S \in F_n$. Let us build a function C(.) defined from $F_n$ to $P(X_n)\setminus\emptyset$, which associates $S \in F_n$ to $C(S) = G(S, R_S)$. Consistent with Hume’s emphasis on the necessary expression of desires into will, C(.) is evidently a choice function. But, according to D-PROC, $G(S, R_S) = G(S, R_{X_n/S})$, where $R_{X_n/S}$ is the restriction of $R_{X_n}$ over $S$. As a result, for any $S \in F_n$, $C(S) = \{x \in S : x R_{X_n} y, \forall y \in S\}$. There exists a binary relation $R_{X_n}$ over $X_n$ which rationalizes C(.): the choice function C(.) is therefore rational.

2.3. Transitivity as an outcome of a decision process

It has very likely not escaped the reader that, stated as above, the decision process described by D-PROC has no explicit end. We can identify, at step n, $C(X_n) = G(X_n, R_{X_n})$, and assume that if no other $(R_{S_i}, S_i)$ appears at step $i \geq n+1$, this would constitute the agent’s actual choice. But Hume puts forward a more interesting insight into the way such a process comes to an end. In a passage where he stresses the role of the relation of impression, he writes:

“Grief and disappointment give rise to anger, anger to envy, envy to malice, and malice to grief again, till the whole circle be compleated” (THN II: 283; see also DP: 145).

Till the whole circle be completed: Hume’s expression suggests a cumulative process, in which envy, malice, and grief – all the possible emotions linked by resemblance – follow one another again and again, till they do not change anything to our general dispositions, that is, to our desires and volition. From the point of view of the decision process, this means that there exists a step, denoted n, after which the process described by D-PROC brings to the fore all the subsets $S_i$ of $X_n$ which are from now onwards always associated to contextual preferences $R_{S_i}$ rationally equivalent to $R_{X_n/S_i}$, the restriction to $S_i$ of general preferences over $X_n$:

$$\exists n : \forall i \geq n+1, S_i \subseteq X_n \Rightarrow G(S_i, R_{S_i}) = G(S_i, R_{X_n/S_i}) \quad [2.6]$$

From step n on, [2.6] clearly ensures that the condition in UPDATE for the revision of contextual preferences over $S_n$, or for the extension of $X_i$ is not satisfied, provided these $S_i$’s are subsets of $X_n$: in Hume’s words, the whole circle is completed. The stabilization property involved in the existence of such a n in [2.6], henceforth expresses some natural disposition of our mind. We are naturally inclined to stabilize our preferences, and after n is reached, it is obvious that the choice over $X_n$ can no more be challenged, but by some event which would introduce exogenously a new $(R_{S_i}, S_i)$ where $S_i$ is not included in $X_n$. Proposition 2 below,
then shows that, when the stabilization condition [2.6] is satisfied, the general preference relation \( R_{Xn} \) is transitive\(^1\).

**PROPOSITION 2.** Let \( R_{Xn} \) be a general preference relation stabilized at step \( n \), then \( R_{Xn} \) is a preorder.

**Proof.** If \( R_{Xn} \) is stabilized at step \( n \) then by definition, \( \forall S \subseteq X_n, G(S, R_S) = G(S, R_{Xn/S}) \). As if it were abstract, \( F_n \) can thus be extended to \( P(X) \setminus \emptyset \) without loss of generality. Moreover it follows from UPDATE in D-PROC that \( \forall S, S' \subseteq F_n, S \subseteq S' \) and \( G(S', R_{S'}) \cap S \neq \emptyset \) implies \( G(S, R_S) = G(S', R_{S'}) \cap S \). This property is exactly the so-called condition C4 by Arrow (1959). This condition is in general weaker than the Weak Axiom of Revealed Preference (WARP) or the Strong Axiom of Revealed Preference (SARP)\(^2\). However when, as here, the domain of choice is abstract, Arrow (1959) has shown that C4, WARP and SARP are equivalent, and are necessary and sufficient conditions for a rationalization by a preorder (complete and transitive). Hence \( R_{Xn} \) is complete and transitive. This result runs as follows: i) C is decisive, that is \( C(S) \) is not empty whatever \( S \) belonging to \( F_n \); ii) C being rational (proposition 1, above), it is rationalizable by \( \mu \), the revealed preference relation, defined as in the previous footnote; iii) since \( F_n \) can be considered as abstract, it includes the pairs \( \{x, y\} \), therefore \( \mu \) is complete; \( F_n \) also includes the triplets \( \{x, y, z\} \) and the fulfilment of Arrow’s condition C4 forbids the possibility of \( \mu \) being acyclic, therefore \( \mu \) is transitive; iv) finally since \( \mu \) and \( R_{Xn} \) rationalize the same choice function, they are equivalent from a rationality standpoint; \( R_{Xn} \) is therefore transitive, and, since it is also complete, \( R_{Xn} \) is a preorder. \( \blacksquare \)

**CONCLUSION**

The way Hume’s approach challenges our current conceptions of rational choice needs to be clarified. Contrary to the intuitive understanding of Hume’s separation between passion and reason, it does not mean that there is no ground for rationality: a Humean perspective on decision making does not necessarily take us away from rational choice as completeness of

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\(^1\) Obviously, the resulting preferences \( R_{Xn} \) are path-dependent, just like for Hume, a passionate state would depend on a passionate path.

\(^2\) There are various versions of these well-known axioms. However, they can be stated as follows:

- **WARP:** \( \forall x, y \in X_n, xKy \implies \neg y\mu x \)
- **SARP:** \( \forall x, y \in X_n, xK*y \implies \neg y\mu x \)

where:

- \( K \) is defined by \( \forall x, y \in X_n, xKy \iff \exists S \in F_n: x \in C(S) \) and \( y \in S \cap C(S) \);
- \( K^* \) is the transitive closure of \( K \);
- and \( \mu \) is the revealed preference relation, defined by \( \forall x, y \in X_n, y\mu x \iff \exists S' \in F_n, y \in C(S') \) and \( x \in S' \).
the preference relation, compatibility between preferences and choices, or transitivity of preferences. But it prevents us from considering these types of rationality as pre-requisites, which would participate in the shaping of given preferences and choice over given set and domain of choice. Rationality, in each of these acceptations, should rather be understood as the unattended outcome of a passionate choice process, of the very functioning of our mind. So that when we question the rationality of choice from a Humean standpoint, we do not question it as an assumption, but as a result.

And this makes a significant difference. Still from a Humean point of view, again, failures to rationality cannot be attributed to costly operations of the mind induced by optimisation, which would compromise completeness or transitivity; nor can they be attributed to axiological considerations, which would generate a discrepancy between preferences and choice. There is no cost involved in these necessary manifestations of the mind, and all axiological considerations participate in our desires and preferences, as well as in our volition and choice. It follows that failures to transitivity call for more qualified interpretations.

It may be argued, for instance, that these failures are mostly an artefact, which reflects an external observer standpoint: non-rationality would then be a consequence of the observer’s opinion about what should be the agent’s proper set and domain of choice. But in other cases, these failures may be explained by some exogenous event which would suddenly interrupt the process of choice; or by special circumstances which would prevent the normal functioning of the mind, which entails a stabilization of preferences.

Rationality as a consequence, instead of rationality as an assumption: this leads us to acknowledge the importance of the break introduced by the founders of marginalism, especially Jevons (1871), within the slowly evolving flow of philosophy of action. After Jevons, indeed, we have adopted this counter-intuitive intellectual habit to consider a set of choice and a preference relation over it (or a utility function) as given. Following Jevons’ pioneering work, it helped his successors constructing standard modern decision theory under certainty. However, now that this construction is enough worked out to show us both its strength and its vulnerability, we are urged to question its foundations: not in order to reject the rationality involved in given preferences and sets of choice, but to explain it as the possible result of the same mental process which gives birth to preferences and choice. Such an ambition can be found in Hume’s legacy to the theory of decision.
REFERENCES


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