

Pleasure and belief in Hume's decision process

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1. INTRODUCTION

It is currently acknowledged, among Hume scholars interested in modern philosophy of action, that his theory of the motives to action was some kind of belief-desire theory – which refers to what is today called "folk psychology". Of course, the relative parts of belief and desire, as well as their respective contents, have been subject to constant discussion[1]. But the framework of the theory has not been seriously threatened. For economists interested in decision theory, this is also quite familiar: it is clear that we have a belief-desire theory as well, in which "belief" stands for something like probability, and "desire" for preferences.

In this respect, we should have some good argument favouring the idea that, for instance, the expected utility theory is Humean, just because the Savageian "principle of the sure thing" achieves a separation between the world of belief (where reason is required to tell us if such belief is true or false) and the world of desire (where reason alone is impotent). In the same way, the role played by pleasure and pain within Hume's works shouldn't seem so odd according to usual standards of decision theory – provided we view them with sufficient indulgence: at least at first sight, this role seems rather similar to the one played by utility. We

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[1] For instance, a standard interpretation which allows numerous variants (see Jonathan Harrison 1976, Michael Smith 1987, or Elizabeth Radcliffe 1999) asserts that action is motivated by the pair belief-desire, and that belief alone is insufficient to have the same effect. Other commentators, on the contrary, argue that some beliefs are motives to action (Ingmar Persson 1997), or that belief (or, at least, believing) is in some way similar to passion (Barry Stroud 1977, pp. 160 *sqq*.).

know that this last can be considered, independently of its possible content, as a representation of preordered and continuous preference and, consequently, of the possible choice function that it would rationalize (that is, of the choice function whose selected elements correspond to the best elements with respect to the previous preference). As a consequence, it seems possible to argue that pleasure and pain, in a Humean framework, *represent* desire and will, just like utility represents preferences and choices.

Now, both pleasure and belief were minor issues in our previous article on David Hume's theory of choice (M.-A. Diaye and A. Lapidus 2005a). In this paper, we discussed the rationality of a Humean decision process without explicit references to the role of either pleasure or belief. Drawing on Hume's writings, we built there an algorithm of choice called D-PROC. From step to step, in D-PROC, the set of choice X, the domain of choice F (constituted by a set of non-empty subsets S of X, interpreted as "contexts of choice", or "opportunity sets", or "budgets", or "consumption sets", according to alternative terminologies), along with the related contextual preferences R_S (over S), general preferences R (over X), and choice functions C(S) (defined from F to P(X) the set of all subsets of X), are revised, allowing for rather general conditions. However, D-PROC displays, as a possible outcome, *rationality* in two different senses (the second one being stronger than the first), which seem to grasp the intuitive meanings of the word in standard theory of choice under certainty[1]. The first kind of rationality which constitutes a possible outcome of D-PROC is the *rationality of the choice function* (see proposition 1 in Diaye and Lapidus 2005a, p. 105), that is, the consistency between choices and preferences in the sense where an agent, alternatively facing various contexts of choice S, would always choose what he prefers according to his or her general preferences R. More formally, the rationality of a choice function C(S) means that this latter can be rationalized by a binary relation R over X, such that for all S belonging to F, the set of optimal elements of S with respect to R being $G(S, R) = \{x \in S : xRy, \forall y \in S\}$, $C(S) = G(S, R)$. The second kind of rationality which might be produced by D-PROC is the *rationality of the preference relation* (see proposition 2 in Diaye and Lapidus 2005a, p. 106), which means that preferences are preordered. In other words, R is complete, (that is, for all x, y belonging to X, x R y or y R x) and transitive (that is, for all x, y, z belonging to X, x R y and y R $z \Rightarrow x$ R z). Rationality of the preference relation appears as the effect of a stabilization property of the decision process, when the agent is no more inclined to revise his or her preferences.

A purpose of this paper is therefore to reintroduce pleasure and belief in their due place, in what aims at being a Humean theory of decision. The case of pleasure deserves

[1] For a more detailed discussion, see Diaye and Lapidus 2005a, pp. 91-3.

special consideration (**section 2**). In the following, we support the idea that Hume was in some way – evidently different from Bentham's or Jevons' way – a hedonist. However, the emphasis should be put less on what is shared by Hume and Bentham, than on the specific kind of hedonism encountered in Hume's writings (chiefly the *Treatise of Human Nature* (1739-40), the *Enquiry Concerning the Principles of Morals* (1751), the *Dissertation on the Passions* (1757), or in some of his *Essays* (1777)):

- Pleasure does represent desire but, in contrast to utility in its relation to preferences, it is because the latter (desire) is generated by the former (pleasure) – and not the reverse.
- Again in contrast to utility in its relation to preferences, the order between goods which is reflected on desires can move away from the relation between goods generated by pleasure. In other words, whereas the goods that grant us the greatest utility are also those which we prefer, what pleases us the most is not necessarily what we desire the most[1].

The reason for such a difference with the usual modern approach lies in the mental process that Hume puts to the fore in order to explain the way by which pleasure determines desires and volition. And here, we meet again the other missing element from our previous picture of Hume's theory of choice – belief (**section 3**). Belief helps to make the crucial difference that Hume establishes between two different meanings of "pleasure", as an *impression* and as an *idea*. Whereas pleasure is primarily an impression of sensation, it takes place in the birth of passions as reflecting an idea of pleasure, whose "force and vivacity" is precisely a "belief". Now, such a belief is transferred to the direct passions[2] of desire or volition which come immediately before action. This has a puzzling consequence: when we speak of "belief" from a Humean point of view, our use of the word is rather different from the one usually acknowledged, either in current language or in economic decision theory, since it is no more exclusively related to decision under risk or uncertainty, but it might also express, for instance, the discounting of future goods or the lack of discrimination between different elements of the choice context. The latter is explored within a formal framework, and it is shown that the relation of pleasure is transformed by belief into a class of relations of

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1 Hume expresses this idea by writing, for instance, that a "trivial good [that is, a lesser pleasure; M;-A.D and A.L.] may, from certain circumstances, produce a desire superior to what arises from the greatest and most valuable enjoyment". (Hume 1739-40:, II, p. 416)

2 According to Hume, the "direct passions" are *joy* and *grief*, *hope* and *fear*, *desire* and *aversion*, and *will* or *volition*. They "arise immediately from good or evil, from pain or pleasure" (Hume 1739-40, II, p. 276). The direct passions in relation to decision theory are discussed in Diaye and Lapidus 2005a, pp. 93-6, and in Lapidus 2000, pp. 12-20. But for a more general account, see Páll Árdal 1966, chapters 1, 2, and 5.

desire, among which at least one is a preorder. This allows establishing a link between the pleasure-belief operation and the Humean decision process D-PROC described in Diaye and Lapidus 2005a: pleasure and belief can express themselves through contextual preferences at each step of a decision process which, when completed, becomes rational.

2. PLEASURE AS AN IMPRESSION

2.1. The Impression of Pleasure: A Pre-Benthamite Analysis

Like Bentham, Hume might be viewed as a hedonist when he claimed, nearly in the same terms as his follower, that "[t]here is implanted in the human mind a perception of pain and pleasure as the chief spring and moving principle of all its actions" (Hume 1739-40, I, p. 118)[1] or, in the second *Enquiry*, when he imagined the successive questions which might be asked to a man to understand what motivated him to do some physical exercise (Hume 1751, p. 293). The answers went on, from health to occupation, then to money. But two typical answers only could terminate the sequence: the avoidance of *pain* (an "ultimate end", said Hume) and the search for *pleasure* ("beyond this it is an absurdity to ask for a reason"). Of course, Hume did not enter into as many details as the author of the *Introduction to the Principles of Morals and Legislation* did[2], but we have evidence that he had some rough intuition of pleasure as a scalar magnitude (let us denote it by p), non-negative ($p \geq 0$) when it concerns pleasure strictly speaking, and negative ($p < 0$) in the case of pain. This magnitude depends on:

a) the context of choice S which our passions lead us to consider, and which is here supposed to be included in $\mathbb{R}^n_+$, where n stands for the number of objects;
b) the element x belonging to S which gives rise to pleasure or pain (x is a vector of quantities; when comparing two vectors $x = (x^1,\ldots,x^n)$ and $y = (y^1,\ldots,y^n)$ belonging to S (subset of $\mathbb{R}^n_+$), we will state $x \geq y$ if $x^i \geq x^i$ for any i = 1,…,n, with at least one strict inequality, and $x > y$ if $x^i > x^i$ for any i = 1,…,n);
c) and our emotional state, typically illustrated by what Hume calls the 'degree of violence' of the passions denoted v and belonging to $\mathbb{R}_+$.

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1 Compare to the well-known sentence by which Bentham began the *Introduction to the Principles of Morals and Legislation*: "Nature has placed mankind under the governance of two sovereign masters, *pain* and *pleasure*" (Bentham 1789, p. 1).

2 Bentham described in chapter IV of his work (Bentham 1789, pp. 29-30) some elementary arithmetic operations which allow the calculation of a quantity of pleasure associated to an act on the basis of its "intensity", "duration", "certainty or uncertainty', and "propinquity or remoteness".

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On the basis of the above points a), b), and c), we can write the real-valued function which determines pleasure as:

$$p = p_S (x, v) \qquad [2.1]$$

We will discuss hereafter the variables involved in [2.1] and the properties of the function of pleasure p_S.

It is obvious, from a Humean point of view, that the objects which are presumed to provide us pleasure or pain do not exist for us by the sole virtue of our reason and independently of our perceptions. They come to our mind with other objects as supports of our way to live the pleasure or the pain (*i.e.*, in Hume's words, our 'passions', 'emotions', or 'affections') that they are supposed to give us. This contextual existence of objects, expressed by S, linked to an emotional state *v*, is introduced by Hume as a consequence of the inability of reason alone to let us know the objects that would potentially be submitted, afterwards, to our affections:

> "Where the objects themselves do not affect us, their connexion can never give them any influence; and 'tis plain, that as reason is nothing but the discovery of this connexion, it cannot be by its means that the objects are able to affect us" (Hume 1739-40, II, p. 414).

So that a possible disagreement with a formulation like [2.1] might arise not from the contextual existence of objects which may give us pleasure or pain, but from the nature of the magnitude which represents pleasure, and from the part played by our emotional state. These two questions will be dealt with successively.

For some commentators, indeed, a difficulty might arise, either from an irreducible heterogeneity of pleasures (see Pierre Demeulenaere 1996, pp. 36-7) which would render them definitely incomparable, or from the very nature of the part played by pleasure in the decision process. For instance, Norman Kemp Smith (1941, p. 164) denied, in his influential book, that Hume was a hedonist, and he considered pleasure and pain as efficient causes of action (among others), but not as final causes. It's a fact that we can find (more often in the *Treatise* than in the *Enquiries* or in the *Dissertation*) some warnings, like the following, which take us away from an overly Benthamite reading:

> "[U]nder the term *pleasure,* we comprehend sensations, which are very different from each other, and which have only such a distant resemblance as is requisite to make them be expressed by the same abstract term" (Hume 1739-40, III, p. 472).

But such warnings are usually subordinated to another purpose. For instance, the latter concerns the explanation of the differences between moral virtues, so that it is not obvious that it would be relevant in dealing with the motives to action. On the contrary, even the sentence which follows the above quotation, among many other examples, seems to announce Bentham's future position:

> "A good composition of music and a bottle of good wine equally produce pleasure; and what is more, their goodness is determin'd merely by the pleasure." (*Ibid*.).

Although the proximity between Hume and Bentham with regard to the determination of individual behaviour might be challenged from several other points of view – and we will challenge it too – we will accept for the time being the principle of the determining role of pleasure and of its measurability. From this point of view, it seems quite obvious that, provided the elements of S (included in $\mathbb{R}^n_+$) are defined correctly, $p_S(x, v)$ is monotonously increasing in x:

$$\forall\, x, y \in S, \text{ if } x > y \text{ then } p_S(x, v) > p_S(y, v) \qquad [2.2]$$

Nonetheless, the question of the emotional state, expressed by v in [2.1], will move us apart from an overly Benthamite interpretation.

2.2. Happiness and the Pleasure-Response

It would be an oversimplification to argue that Hume's analysis of the impression of pleasure gives rise to an expression of pleasure within a specific context as depending only on the quantities of the goods concerned. In contrast to Bentham (although chapter VI of the *Introduction to the Principles of Morals and Legislation* (Bentham 1789) might be used in favour of a more qualified reading), Hume considered that the pleasure-response given by such good also depends on our emotional state, exemplified by the degree of violence of the passions. The *Treatise*, like the *Enquiries*, is not as explicit on this relation as one would have expected it to be. A clearer insight of the question is given at the occasion of discussions about the link between pleasure and happiness[1], in two essays[2] that Hume first published in 1742 and 1752 respectively.

Indeed, it is neither in the *Treatise* nor in the *Enquiries*, but in the essay on the "Refinement in the Arts"[3] that Hume introduces the elements whose interaction determines happiness:

> Human happiness […] seems to consist in three ingredients; action, pleasure, and indolence: And though these ingredients ought to be mixed in different proportions, according to the particular disposition of the person; yet no one ingredient can be entirely wanting, without destroying, in some

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1 On the role of happiness (as distinct from both interest and desire) in the valuation of individual welfare from a Humean point of view, see Lapidus 2010.

2 When Hume acknowledged that the *Treatise* did not receive the attention and success that it deserved, he published and revised, up to the posthumous edition of 1777, a collection of short contributions entitled *Essays Moral, Political and Literary*, where he tried to make available to his reader some essential features of his more academic works.

3 For its first publication in the *Essays*, in 1752, this text was entitled "Of Luxury".

> measure, the relish of the whole composition. (Hume 1777, Of Refinement in the Arts [1752], pp. 269-70)

At first sight, this only seems to mean that pleasure is by itself insufficient to give rise to happiness. But the subsequent discussion shows that Hume's position is not so obvious, since "action" and "indolence" do not stand at the same level as "pleasure". If happiness does depend on action and indolence, it is because they influence our response, in terms of pleasure, to any solicitation of our senses. For instance, when the proportion excessively favours action, the "quick march of the spirits, which takes a man from himself, and chiefly gives satisfaction, does in the end exhaust the mind" (*Ibid*., p. 240). By contrast, when indolence predominates, it prompts "a languor and lethargy, that destroys all enjoyment" (*Ibid*.). Now, action and indolence were already related to the degree of violence of the passions in Hume's essay on "The Sceptic"[1]:

> To be happy, the *passion* must neither be too violent nor too remiss. In the first case, the mind is in a perpetual hurry and tumult; in the second, it sinks into a disagreeable indolence and lethargy. (Hume 1777, The Sceptic [1742], p. 167)

This gives sense to indolence and action. They respectively correspond to two mental states, "lethargy" and "tumult", and come along with two states of the passions, the one remiss, and the other violent.

Although Hume's main purpose in the above quoted passages from both the Refinement in the Arts and from the Sceptic was to explain what contributes to happiness, he also made obvious that he did not view the pleasure generated by an object as independent from the emotional state of the individual – that is, of the degree of violence of the passions which govern him. This gives meaning to the representation of a Humean function of pleasure, as suggested in [2.1]. However, Hume went further, asserting two conjectural properties to this function.

The *first property* concerns the degree of violence of the passions. In Hume's works, this question is usually related to intertemporal decision: the preference for present expresses what Hume called the "degree of violence of passions" (see Lapidus 2000, pp. 45-9; Davis 2003; Palacios-Huerta 2003; Lapidus 2010). When the time preference is low, the passion is said to be "calm" in the *Treatise* (see, for example, Hume 1739-40, II, p. 418), the

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[1] First published in 1742, this essay is one of the four texts ("The Epicurean", "The Stoic", "The Platonist", "The Sceptic") in which Hume presents "sects, that naturally form themselves in the world, and entertain different ideas of human life and of happiness" (Hume 1777, The Epicurean [1742], p. 138 n. 1). The question of knowing whether this essay corresponds to Hume's position is still discussed. We follow here John Immerwahr's interpretation according to which, while none of the four essays fits perfectly with the theses developed in the *Treatise*, "The Sceptic" comes closest to them (Immerwahr 1989). The main differences noted by Immerwahr concern the possibility of an emotional control which could lead to happiness.

Dissertation (Hume 1757, p. 162) or the second *Enquiry* (Hume 1751, p. 239); and when it increases, the passion becomes more violent. But such a calm passion might also be viewed as the intermediate state described in the Sceptic, between the "remiss" and the "violent" passion. According to the above mentioned passages from the *Essays*, we might conclude that for Hume, the highest pleasure-response is produced by a calm passion. Allowing that the pleasure function $p_S(x, v)$ is differentiable in v and denoting $\hat{v}$ the degree of violence of the calm passion, this means that the partial derivative of p_S with respect to v is positive as far as the passion is remiss ($v < \hat{v}$), negative when it is violent ($v > \hat{v}$), and zero when it is calm ($v = \hat{v}$). That is to say, given S and x (belonging to S), $p_S(x, v)$ is at a maximum for a calm passion:

Property 1: Maxima of the pleasure function with respect to the violence of passions
Given S and x in S, $\hat{v} \equiv \underset{v}{ArgMax}\, p_S(x,v)$ [2.3]

The *second property* concerns the pleasure-response to the variations of the quantities x. It can be inferred from a passage of the *Treatise* (Hume 1739-40, I, p. 141-2) in which Hume compared the differences of the impressions between two pairs of amounts of money, that this difference is meaningful. In Hume's example, it means that the difference in the impressions concerning three and two guineas, on the one hand, is greater than the difference in the impressions concerning one thousand and nine hundred ninety-nine guineas, on the other hand.

- Generalizing Hume's position, a first conclusion of this example is that for any v and for any (x, y) and (z, w) belonging to S×S,

  $p_S(x, v) - p_S(y, v) \geq p_S(z, v) - p_S(w, v)$ or $p_S(z, v) - p_S(w, v) \geq p_S(x, v) - p_S(y, v)$

  Extending this assumption to the differences between pleasures generated by different values of v, and denoting $\Delta p_S(x, y; v, v') = p_S(x, v) - p_S(y, v')$, the previous condition can be rewritten:

  $\Delta p_S(x, y; v, v') \geq \Delta p_S(z, w; v'', v''')$ or $\Delta p_S(z, w; v'', v''') \geq \Delta p_S(x, y; v, v')$ [2.4].

  Because pleasure differences are meaningful (see equation [2.4]), the Humean pleasure function $p_S(.)$ can be considered as a cardinal function with normalized zero, that is, a function defined up to a positive linear transformation g ($g(p_S) = a\, p_S$, with $a > 0$).
- Moreover, an obvious second conclusion is that if, as Hume argued, the difference of the impressions between three and two guineas is greater than the corresponding difference between one thousand and nine hundred ninety-nine guineas, $p_S(x,v)$ is concave with respect to x.

The second property hence comes as follows:

Property 2: Cardinality and concavity of the pleasure function

$p_S(x, v)$ is

i) cardinal with normalized zero; [2.5]

ii) concave with respect to x.

Now, these two properties do not play the same part within Hume's analysis. Property 1 depends on a special Humean conjecture on the calm passion, namely the idea that there exists an emotional state that would not only lead us to the decisions that fit our interest the best, but would also provide us the greatest pleasure for each object. However, although this conjecture is crucial to understand Hume's practical conception of interest and happiness (see Lapidus 2010), and justifies the importance that he granted to the calm passions in many fields of the social life (see John Immerwahr 1992), it seems far less important from the more general point of view of the decision theory which stems from his works. On the contrary, property 2 seems to grasp the very nature of the sensation of pleasure involved in this decision process.

2.3. Indiscrimination and the Correspondence of Pleasure

However, before going any further, let us stress a formal consequence of the mere existence of $p_S(x, v)$, even in the case where it does not satisfy properties 1 and 2. It is well known that if a function like the function of pleasure p_S is defined on the context of choice S as above, and if S has some convenient property (for instance, if S is countable), it might be viewed as a representation of an underlying binary pleasure relation $L_S(v)$ depending on v. $L_S(v)$ being a non-empty subset of S×S, $(x, y) \in L_S(v)$ reads "given v, x pleases at least as much as y", and can be alternatively and more conveniently written $x\, L_S(v)\, y$. Asserting that the function of pleasure p_S is a *representation* of the pleasure relation L_S amounts to saying that it is an order homomorphism defined from $(S, L_S(v))$ into $(\mathbb{R}, \geq)$:

$$\forall x, y \in S,\ x\, L_S(v)\, y \Leftrightarrow p_S(x, v) \geq p_S(y, v) \qquad [2.6]$$

where $L_S(v)$ is a preorder on S; that is $L_S(v)$ satisfies the properties of completeness [2.7] and transitivity [2.8]:

$$\forall x, y \in S,\ x\, L_S(v)\, y \text{ or } y\, L_S(v)\, x \qquad [2.7]$$

$$\forall x, y, z \in S,\ x\, L_S(v)\, y \text{ and } y\, L_S(v)\, z \Rightarrow x\, L_S(v)\, z \qquad [2.8]$$

In most cases, when Hume refers to pleasure as a magnitude, he gives us evidence that we are in a situation which allows [2.6] to be true. Hence on the one hand, arguing that in a given context of choice we are entitled to associate a given amount of pleasure (expressed by a real number) to each element of S, amounts to saying that between two elements of S, we are always able to determine which one pleases us the most (completeness). On the other hand if a first element pleases us at least as much as a second, and if this second element pleases us at least as much as a third, then the first element pleases us at least as much as the third (transitivity). This is all the more so that property 2 above is satisfied, and that pleasure appears as a cardinal magnitude. But in some other cases, although his vocabulary remains the

same, Hume seems to express something rather different, so that the existence of $p_S(x, v)$ and, as a result, the existence and the properties of $L_S(v)$ become much more disputable.

An interesting example can be found in a previously quoted passage of the *Treatise*, at the end of the section of the first Book of the *Treatise* dedicated to the "probability of causes", in which Hume discusses "minute differences" between probabilities. He imagined what we would call a "lottery" (x_1, x_2; $prob_1$, $prob_2$), where $prob_1$ = 10000/20001 and $prob_2$ = 1-$prob_1$ = 10001/20001:

> "My […] reflection is founded on those large probabilities which the mind can judge of, and the minute differences it can observe betwixt them. When the chances or experiments on one side amount to ten thousand, and on the other to ten thousand and one […] it is plainly impossible for the mind to run over every particular view, and distinguish the superior vivacity of the image arising from the superior number, where the difference is so inconsiderable." (Hume 1739-40, I, p. 141).

Hume's argument on the impossibility for the mind to "run over every particular view, and distinguish the superior vivacity of the image arising from the superior number" is based on a fundamental mechanism, first built in the context of the "probabilities of chances", and then extended to the "probabilities of causes". They respectively correspond roughly to the *a priori* calculus of chances, usually illustrated by examples drawn from gambling, and to the statistical judgement, which infers predictions on the class to which belongs such character from past observations concerning the distribution of characters between classes.

It was in order to give new evidence which would favour his argumentation that Hume shifted from the analysis of relations concerning ideas towards such relations concerning impressions of sensation (pleasure and pain) or of reflection (passions). "We have a parallel instance in the affections", he said (Hume 1739-40, I, p. 141). Instead of "numbers" referring to probabilities, we now have "numbers" representing quantities of impressions which, in turn, refer to quantities of objects. Hume then suggested a simple arithmetic of passions, in which a large "compounded" passion comes from "a great number of weaker passions, derived from a view of each part of the object" (*ibid.*), and pointed out a principle similar to the one which prevails in the probability of causes:

> "Yet nothing can be more certain than that so small a difference [Hume's example concerns the desire for one pound more out of one thousand; M.-A.D. and A.L.] wou'd not be discernible in the passions, nor cou'd render them distinguishable from each other." (Hume 1739-40, I, p. 141).

As a result, Hume argued, although "[t]he mind can perceive, from its immediate feeling, that three guineas produce a greater passion than two", it cannot perceive any distinguishable sensation between one thousand guineas and nine hundred and ninety-nine guineas (Hume 1739-40, I, p. 141-2).

A seemingly obvious interpretation leads one to infer from the above passage that Hume assumed an initially positive decreasing marginal pleasure, which becomes constant from the point where it is equal to zero. This obvious interpretation supports, as above (see

property 2, [2.5]), the idea that Hume had in view what we would call a "concave" pleasure function. However, there is something more in this example. Hume also argued that we are usually facing what is today called "indiscrimination problems" whose weight is all the more important when we are dealing with smaller relative differences between the objects compared to each other (Lapidus 2000, pp. 49-51). The question of indiscrimination is now familiar in decision theory, at least since R. Luce (1956). From the point of view of Hume's example, an important consequence of the indiscrimination problem on which he laid emphasis would be that, for instance, 1000 guineas would provide neither more nor less pleasure than 999, 1001 than 1000, 1002 than 1001, etc… whereas 1009 guineas would give more pleasure than, let us say, 999. But such a use of the expressions "more pleasure" or "less pleasure" can only be metaphorical, since in this case, pleasure as a magnitude determined by a pleasure function like in [2.1], might merely not exist. This can be explained more formally.

The pleasure relation $L_S(v)$ can be considered as the union of its asymmetric part $PL_S(v)$ and its symmetric part $IL_S(v)$ of $L_S(v)$, defined as follows:

$$PL_S(v) = \{(x, y) : (x, y) \in L_S(v) \text{ and } (y, x) \notin L_S(v)\}$$
$$IL_S(v) = \{(x, y) : (x, y) \in L_S(v) \text{ and } (y, x) \in L_S(v)\} \qquad [2.9]$$
$$L_S(v) = PL_S(v) \cup IL_S(v) \text{ (whereas } PL_S(v) \cap IL_S(v) = \varnothing)$$

Hume's example of indiscrimination between 999 and 1000 guineas did not challenge the asymmetric part $PL_S(v)$: this latter is clearly transitive, just as it would be if no discrimination problem occurs. But such is not the case with the symmetric part $IL_S(v)$: we may find sequences (x_1, x_2), (x_2, x_3), … $(x_{n-1}, x_n) \in IL_S(v)$ although $(x_1, x_n) \in PL_S(v)$[1]. The property of transitivity [2.8] of the pleasure relation $L_S(v)$ should therefore be restricted to its asymmetric part only, so that it is said to be only "quasi-transitive":

$$\forall\, x, y, z \in S,\ x\ PL_S(v)\ y \text{ and } y\ PL_S(v)\ z \Rightarrow x\ PL_S(v)\ z \qquad [2.10]$$

Since $L_S(v)$ satisfies [2.7] and [2.10], it is a complete quasi-transitive relation. And it is now well known that in this case, no scalar function like $p_S(x, v)$ can represent $L_S(v)$. This is why we argued that the reference to such or such amount of pleasure in order to denote, like in the standard situation of perfect discrimination, the impressions associated to various amounts of goods, can only be metaphorical. It seems that Hume was employing a vocabulary which matches the literary description of a functional relation – the function of pleasure – in a situation where such a functional relation cannot exist.

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[1] For instance, (999, 1000), (1000, 1001), … (1008, 1009) $\in IL_S(v)$, but (999, 1009) $\notin IL_S(v)$ since (1009, 999) $\in PL_S(v)$.

Rigorously speaking, instead of a *pleasure function* $p_S(x, v)$, it is a *pleasure correspondence* $\pi_S(x, v)$ that Hume's example would suggest – that is, a mapping of the context of choice S, into the set of subsets (in this case, closed intervals) of the quantities of pleasure:

$$\pi_S(x, v) = [a_S(x, v), b_S(x, v)] \quad [2.11]$$

(where $x \in S$ and $[a_S(x, v), b_S(x, v)]$ is an interval in $\mathbb{R}$)

[2.11] amounts to saying that the pleasure of x is not a single point, let us say $p_S(x, v)$, but is located within an interval closed by a minimum $a_S(x, v)$ and a maximum $b_S(x, v)$ of pleasure. Let us remember that a closed interval I_1 (included in $\mathbb{R}$) is greater than another closed interval I_2 (also included in $\mathbb{R}$) denoted $I_1 >> I_2$ if any element from I_2 is strictly higher (>) than any element from I_1 : whatever α_1 belonging to I_1 and α_2 belonging to I_2, $\alpha_1 > \alpha_2$. In order to simplify the notation and if there is no risk of confusion, we write $I_1 > I_2$ instead of $I_1 >> I_2$. Henceforth, the pleasure correspondence $\pi_S(x, v)$ in [2.11] represents the pleasure relation $L_S(v)$ in the following way:

$$\forall x, y \in S, x \, IL_S(v) \, y \Rightarrow \pi_S(x, v) \cap \pi_S(y, v) \neq \varnothing \quad [2.12]$$

$$\forall x, y \in S, x \, PL_S(v) \, y \Rightarrow \pi_S(x, v) > \pi_S(y, v) \quad [2.13]$$

[2.12] means that the individual is indifferent in terms of pleasure between bundles x and y if the maximum of pleasure associated with x, $b_S(x, v)$, is between the minimum of pleasure associated with y, $a_S(y, v)$ and the maximum of pleasure associated with y, $b_S(y, v)$. Likewise, according to equation [2.13], x pleases strictly more than y to the individual if the minimum of pleasure associated with x, $a_S(x, v)$, is greater than the maximum of pleasure associated with y, $b_S(y, v)$. Moreover, it seems obvious that the monotony property [2.2] of the pleasure function be extended to the correspondence of pleasure so that, if the bundle x is greater than the bundle y, the lower and the upper bounds of $\pi_S(x, v)$ are greater than the corresponding bounds of $\pi_S(y, v)$:

$$\forall x, y \in S, x > y \Rightarrow a_S(x, v) > a_S(y, v) \text{ and } b_S(x, v) > b_S(y, v) \quad [2.14].$$

3. BELIEF, PLEASURE, AND THE WAY TO ACTION

3.1. A Sophisticated Hedonism

The construction, in contemporary decision theory, of decision patterns based on complete quasi-transitive preferences, seems to open the path to a similar issue concerning Hume's decision process. This issue would have made possible carrying on with the parallel between Hume and Bentham. However, such a path is barred and the parallel with Bentham cannot be continued any longer. To put it briefly, this comes from the divergence between, on the one hand, the pleasure relation $L_S(v)$ or the correspondence of pleasure $\pi_S(x, v)$ which represents it, and, on the other hand, a desire binary relation $R_S(v)$ on S which denotes

individual preferences. In other words, and this time in sharp contrast to Bentham, what pleases us the most from a Humean point of view is usually not what we desire the most. This is not enough to give up the idea that Hume's theory of action comes within *psychological hedonism*, but it is clearly enough to consider it a *sophisticated hedonism*.

This question may be tackled through one of the best known passages of the *Treatise*, in which Hume gave instances of the inability of reason to arbitrate between alternative ends of passions – as long as they are neither "founded on false suppositions" nor choose "means insufficient for the end":

> "'Tis not contrary to reason to prefer the destruction of the whole world to the scratching of my finger. 'Tis not contrary to reason for me to chuse my total ruin, to prevent the least uneasiness of an *Indian*, or person wholly unknown to me." (Hume 1739-40, II, p. 416).

Hume does not mention explicitly here pleasure or pain. Nonetheless, we could conclude from these two provocative examples, systematically quoted in the literature on Hume's theory of morals, that what is *preferred* or *chosen* (Hume's vocabulary is far from definite) in each situation is also what would provide the greatest pleasure (or, rather, the smallest pain): the destruction of the world in one case, my ruin in the other. But the following sentences lead us to qualify this interpretation:

> "'Tis as little contrary to reason to prefer even my own acknowledg'd lesser good to my greater, and have a more ardent affection for the former than the latter. A trivial good may, from certain circumstances, produce a desire superior to what arises from the greatest and most valuable enjoyment." (*Ibid*.).

There is no doubt that the "good" mentioned by Hume stands for "pleasure"[1], so that it is obvious that Hume means that we might as well follow our pleasure or go against it[2]. However, the way Hume illustrates the possible contradiction between pleasure and preferences is far from fully satisfactory:

> "[There is not] any thing more extraordinary in this", Hume continued, "than in mechanics to see one pound weight raise up a hundred by the advantage of its situation" (*Ibid*.).

Although rather obscure, this mechanical analogy leads us to question the nature of the suggested *lever arm*, the "advantage" of the situation, which allows pleasure to determine desire and will on the one hand, and to be ordered differently on the other hand.

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1 In various instances, when discussing the origin of direct passions, Hume was quite specific in pointing out that "good and evil" are, "in other words, pain and pleasure" (Hume 1739-40, II, p. 439). Both the *Treatise* and the *Dissertation* make clear that for Hume, the words "pleasure", "good", "agreement" were synonymous as impressions of sensation, just like "pain", "evil", and "disagreement".

2 Only the *two* first situations of choice among the three involved in this passage ((1) the destruction of the world; (2) my ruin; (3) my lesser good) are usually discussed in the literature. Cass Weller 2004 is a notable exception, since he also discusses the third situation.

The location of the answer is quite familiar to Hume scholars. Indeed, it is well-known that in situations of choice, the pleasure which activates the direct passions that are the closest to action (desire, aversion, and will) stands not as an "impression of sensation" (using Hume's vocabulary) but as an "idea" derived from this impression:

> "But pain and pleasure have two ways of making their appearance in the mind; of which the one has effects very different from the other. They may either appear in impression to the actual feeling, or only in idea, as at present when I mention them." (Hume 1739-40, I, p. 118).

Now, it is also well-known from Hume's theory of ideas developed in Book I of the *Treatise* that an idea as such, as a mere conception, cannot cause any action even if its correlate is a very intense impression:

> "'Tis evident the influence of these upon our actions is far from being equal. Impressions always actuate the soul and that in the highest degree; but 'tis not every idea which has the same effect" (*Ibid.*)

Hume thus held that the idea would borrow force and vivacity from the original impression[1] in order to cause action. The pleasure which is related to such good as an original *impression* might thus be very high; but it will not lead on to volition, able to result in action, if the *idea* of this high pleasure is not strong enough. Now, the strength of the idea is precisely what Hume called a "belief": "A LIVELY IDEA RELATED TO OR ASSOCIATED WITH A PRESENT IMPRESSION" (Hume 1739-40, I, p. 96). He explained in detail that our lives would be unbearable if either impressions or ideas alone influence our will (Hume 1739-40, I, pp. 118-9). The "castle-builder" to which he referred frequently, presumably attaches an intense impression of pleasure to the results of his "loose reveries"[2]. But the resulting idea remains an "idle fiction", a fancy of his imagination, which would never influence his will. On the contrary,

> "[…] the ideas of those objects which we believe either are or will be existent, produce in a lesser degree the same effect with those impressions, which are immediately present to the senses and perception. The effect then of belief, is to raise up a simple idea to an equality with our impressions, and bestow on it a like influence on the passions. This effect it can only have by making an idea approach an impression in force and vivacity." (Hume 1739-40, I, p. 119).

Belief hence appears as the appropriate candidate for the 'lever arm' for which we were searching. Its birth is a complicated operation – for which Hume remained so unsatisfied that he devoted half of the Appendix to the *Treatise* to it, published jointly to Book III in

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[1] The *Enquiry* is not as clear as the *Treatise* in considering that an impression "*communicates a share of its force and vivacity*" (Hume 1739-40, I, p. 98). It only states that the force of a conception is "derived" from a present impression (Hume 1748, p. 54). The thesis of an evolution on this point from the time when Hume was writing Book I of the *Treatise*, to the Appendix published one year after the two first books, and to the *Enquiry on Human Understanding* was brilliantly supported by Francis Dauer 1999.

[2] Hume 1739-40, I, p. 97. The example of the "castle-builder" reappears in the Appendix to the *Treatise* (Hume 1739-40, A, pp. 624; 625) where it meets the one of an "enchanted castle" (Hume 1739-40, A, p. 625), also present in the *Enquiry* (Hume 1748, p. 50).

1740, one year after Books I and II, which involves *custom*, *general rules*, and *natural relations*.

It can be observed that the numerous examples, scattered in Hume's writings, which show a possible discrepancy between pleasure and desire or will, usually refer to "custom and practice […] [which] must certainly […] guide us, by means of general establish'd maxims, in the proportions we ought to observe in preferring one object to another" (Hume 1739-40, II, p. 294). Now, "custom" is not a trifling word in Hume's system. What he called indifferently "custom" or "habit"[1] is a touchstone in the explanation of inductive inference, of which it is well-known that he claimed it could not be rationally justified. It is a touchstone because:

1. it preserves the reasonings from experience from sceptical doubts, and goes over them, henceforth allowing this "science of man" that Hume was willing to build[2];
2. as the "great guide of human life", as Hume (1748, p. 44) called it, "custom" is also a condition for goal-oriented action and for the natural sciences[3].

On this account, custom is not an external principle, sociological for example, but a principle of the mind that, jointly with the principle of experience, transfers the past to the future without delay or deliberate effort ("[t]he custom operates before we have time for reflexion", Hume (1739-40, I, p. 104) said), and makes us form ideas concerning matters of fact in a more intense manner:

> "Experience is a principle which instructs me in the several conjunctions of objects for the past. Habit is another principle which determines me to expect the same for the future; and both of them conspiring to operate upon the imagination, make me form certain ideas in a more intense and lively manner than

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1 However, one may observe a slight difference, "custom" denoting at times the process of which "habit" is the result.

2 This point is presented, for instance, in section V of the first *Enquiry*: "Nor need we fear, that this philosophy [academic or sceptical philosophy], while it endeavours to limit our enquiries to common life, should ever undermine the reasonings of common life, and carry its doubts so far as to destroy all action, as well as speculation. Nature will always maintain her rights, and prevail in the end over any abstract reasoning whatsoever. Though we should conclude, for instance, as in the foregoing section [on sceptical doubts concerning the operations of the understanding], that, in all reasonings from experience, there is a step taken by the mind, which is not supported by any argument or process of the understanding; there is no danger, that these reasonings, on which almost all knowledge depends, will ever be affected by such a discovery. If the mind be not engaged by argument to make this step, it must be induced by some other principle of equal weight and authority; and that principle will preserve its influence as long as human nature remains the same." (Hume 1748, pp. 41-2).

3 "It is that principle alone, which renders our experience useful to us, and makes us expect, for the future, a similar train of events with those which have appeared in the past. Without the influence of custom, we should be entirely ignorant of every matter of fact, beyond what is immediately present to the memory and senses. We should never know how to adjust means to ends, or to employ our natural powers in the production of any effect. There would be an end at once of all action, as well as of the chief part of speculation." (Hume 1748, pp. 44-5; see also *ibid*., p. 55).

> others which are not attended with the same advantages. Without this quality, by which the mind enlivens some ideas beyond others (which seemingly is so trivial, and so little founded on reason), we could never assent to any argument, nor carry our view beyond those few objects which are present to our senses." (Hume 1739-40, I, p. 265).

This feature is the effect of custom. It does not only make us pass from such past or present impressions or ideas to such future ideas but it also endows these ideas with more or less force or vivacity where certain demonstrative knowledge is missing. In other words, it makes them a *belief*.

3.2. The Lever-Arm: Belief, and Pleasure as an Idea

For Hume, belief lays on some intermediary area between impressions, to which it is said, at least in the *Treatise*, that it borrows force and vivacity (Hume 1739-40, I, p. 98), and ideas standing as mere conceptions, for which force and vivacity are meaningless. If we trust Hume's vocabulary, my belief can be more or less vivid, like an impression. But it does not mean that it is an impression, since it aims at being true or false, and not at being satisfied, let us say, like a desire. Nor is it an idea since, for instance, my belief that it will rain this afternoon might be vivid or weak, or even true or false: it does not change anything to my mere idea of a rainy afternoon[1]. If I take an umbrella, it is not because of this idea of a rainy afternoon; and it is even not because of my belief that it will rain alone: it is because of a) my *desire* to remain dry, and b) my *belief* that the afternoon will be rainy, and, of course, c) my *belief* that an umbrella will give me appropriate shelter.

The vocabulary used by Hume to denote a belief[2] – "vivacity", "vividness", "strength", "force", etc… – nonetheless remains metaphoric and unclear concerning the nature of what is denoted. Moreover, it looks like multi-purpose vocabulary. The same words are used, (i) to establish a distinction between impressions and ideas (Hume 1739-40, I, p. 2; p. 19), (ii) to explain what is transferred from an impression to produce a belief (*Ibid*., pp. 96; 153), (iii) to contrast a belief and a fiction of the imagination (Hume 1739-40, A, p. 629), (iv)

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1 Since it is not an idea, belief is all the more so not some kind of new idea, attached to a former conception. Hume clearly held that there is no novelty in belief (see, for instance, Hume 1739-40, A, p. 626; 1740, p. 653), and that it only consists in a peculiar conception, stronger and more vivid (Hume 1739-40, I, p. 96; 1748, p. 49).

2 In the *Abstract*, Hume recapitulated the synonyms he used to characterize a belief: "He [Hume] calls it sometimes a *stronger* conception, sometimes a more *lively*, a more *vivid*, a *firmer*, or a more *intense* conception." (Hume 1740, p. 654). A similar enumeration can be found in the Appendix to the *Treatise* (Hume 1739-40, A, p. 629), and later in the first *Enquiry*: "I say then, that belief is nothing but a more vivid, lively, forcible, firm, steady conception of an object, than what the imagination alone is ever able to attain." (Hume 1748, p. 49).

to distinguish memories and imagination (Hume 1739-40, I, p. 9; p. 85). And, (v) when our belief is as "vivid" as possible, it allows us to reach the highest reachable level of certainty concerning matters of fact. But, facing a demonstrative knowledge which entails an absolute certainty like, for instance, when we say that the three angles of a triangle equal two right angles (Hume 1739-40, I, p. 166), we do not encounter anything which corresponds to this kind of vivacity. Still more, as Kemp Smith (1941, pp. 169-73) first pointed out, (vi) a similar transfer mechanism of vivacity is implicated in the working of sympathy (Hume 1739-40, II, p. 317).

These problems are rather familiar to most modern commentators of Hume, and it is not obvious that the same notions of "force and vivacity" be involved in each of these six cases. For instance, Trudy Govier (1972) distinguishes the terms denoting "force" from those denoting "vivacity", and attributes the former to the difference between conception and belief, and to the difference between imagination and memories, whereas the latter corresponds to the difference between idea and impression. Wayne Waxman (1993) opposed two axes of differentiation of the perceptions, the first one in terms of the qualities of these perceptions, and the second in terms of the qualities of our consciousness of these perceptions – which allow him to bring together the "vivacity" of a belief and "verisimilitude", that is, to some sense of truth. And Francis Dauer (1999) associates the distinction between impressions and ideas to a sense of presentedness, memory and ideas to a sense of recognition, and belief and conception to an adverbial modification of conceiving (a "belief that p" meaning "believingly conceiving p"). We will enter neither the debate on the meaning of the terms used by Hume to characterize a belief, nor the associated debate on the consistency of Hume's theories of impressions, ideas, and belief. But the very existence of these debates shows that, in spite of what Hume sometimes seemed to argue, "force and vivacity" are technical terms, which entertain only loose relations to usual language. And of course, the same can be said about "belief". In Book I of the *Treatise*, Hume regards "belief" as self-explainable. But the Appendix reconsiders the question, so that it has become clear that "belief" also be a technical term, not identical to its current meaning, and from an economist's point of view, not identical to its standard interpretation in the theory of decision as a probability.

This rather unusual meaning of "belief" might be reached by focusing on the way it is built up, when the force and vivacity of an impression are transferred to an idea. This is the role of what Hume called the "natural relations": *resemblance*, *contiguity*, and *causality* (see Hume 1739-40, I, pp. 10-3; 1748, pp. 23-4). They are so fundamental to the operations of our minds that, commenting on his own previous work, he was to refer to them as "the cement of the universe" (Hume 1740, p. 662). Fundamental, these three relations are not, however, equally decisive. Hume devoted the whole Section IX of Book I, Part III of the *Treatise* to convince his reader that whereas causality establishes a sort of conduit that allows the vivacity

of a present impression to be transferred to an idea[1], resemblance or contiguity alone cannot constitute such a conduit, though they can enhance or mitigate sometimes spectacularly the force and vivacity of the correlated idea[2]. It should be stressed that these natural relations only *associate* ideas. They are distinct from the relations of same names presented as "philosophical relations" (Hume 1739-40, I, pp. 69-78), which depend on the particular circumstance by which we deliberately *compare* two ideas "without a connecting principle" (*Ibid.*: 14). On the contrary, with regard to natural relations, our mind slides from one object to another without any deliberate effort. A belief comes to our mind like this: custom links an idea to an impression by way of the natural relations of causality, resemblance, or contiguity[3]; and this idea is given a part of the force and vivacity of the original impression[4].

We already noticed that the *lever arm* which, for Hume (1739-40, II, p. 416), allows a "one pound weight" (a lesser impression of pleasure) to "raise up a hundred" pounds (a greater desire), is a belief. This can be made more explicit. Belief works like a lever arm in that pleasure is applied to it, and according to the resulting vivacity of the idea of pleasure (its length, as a lever-arm), it activates direct passions close enough to action, just like in mechanics a force lifts up a weight. And since belief depends on the natural relations, the structure of pleasure as an impression will be transformed according to these natural relations in order to produce different structures of the direct passions.

From the point of view of the decision process, the lever arm of belief, which transfers pleasure to the direct passions according to its length, raises two different questions.

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1 F. Dauer 1999 described this kind of relation between impressions, beliefs and ideas as a "hydraulic model" which would not survive the *Treatise*, and would be accordingly missing from the *Enquiry*.

2 Hume 1739-40, I, pp. 106 *sqq*. Concerning the asymmetric roles of causality, on the one hand, and resemblance and contiguity, on the other hand, see Harold Noonan 1999, pp. 73-5.

3 The importance of the natural relations from the point of view of Hume's theory of choice was already emphasized when dealing with the connection between the elements of a context of choice (Diaye and Lapidus 2005a, pp. 96-7), and with the "double relation", of impression and of idea, which generates the dynamics of passions (*Ibid*., 100-1; see also Sugden 2005, pp. 115-6, and Diaye and Lapidus 2005b, p. 121).

4 Though belief establishes a link from past to future, it does not amount to a simple partial transfer: the past is also transformed, by means of "general rules" discussed in Book I, Part III, Section XV of the *Treatise* (Hume 1739-40, I, pp. 173-6). These general rules might today appear obsolete. But beyond this little generous appreciation, general rules constitute a two stages device (Gilles Deleuze 1953, p. 30 distinguished "extensive" and "corrective" rules; a similar idea can be found in Thomas Hearn 1970, Marie Martin 1993, Lapidus 2000, pp. 41n, 66-8) which i) allows extending the domain of custom beyond the simple repetition of the past, and ii) gives some principles in order to control this extent. And such characteristics, of course, are far from obsolete.

The first one deals with the passion involved in the distortion of pleasure, at the extremity of a chain of emotions and beliefs. Hume had described the way this chain is constructed, though his aim was, again, to stress the limited role of reason:

> "'Tis obvious, that when we have the prospect of pain or pleasure from any object, we feel a consequent emotion of aversion or propensity, and are carry'd to avoid or embrace what will give us this uneasiness or satisfaction. 'Tis also obvious, that this emotion rests not here, but, making us cast our view on every side, comprehends whatever objects are connected with its original one by the relation of cause and effect. Here then reasoning takes place to discover this relation; and according as our reasoning varies, our actions receive a subsequent variation. But 'tis evident, in this case, that the impulse arises not from reason, but is only directed by it. It is from the prospect of pain or pleasure that the aversion or propensity arises towards any object: and these emotions extend themselves to the causes and effects of that object, as they are pointed out to us by reason and experience." (Hume 1739-40, II, p. 414).

It is easy to understand that the successive beliefs which separate, for instance, my backache from last week end when I tried to cut the grass, and my buying a rowing machine, have, at each step, transferred more or less of the vivacity of the impression of pain that I endeavour to avoid onto the corresponding passion. At the end of the process, my desires and will have been affected in proportion. More formally, this means that the already noted pleasure relation $L_S(v)$ which underlies the correspondence of pleasure $\pi(x, v)$ is transformed by belief into a desire binary relation $R_S(v)$.

The second question raised by the transfer of pleasure to the direct passions deals with the natural relation whose role is favoured in the birth of belief. Three kinds of transformation of the structure of pleasures can be distinguished, according to the corresponding natural relation – causality, resemblance, and contiguity – favoured by the process.

(1). *Causality* is always involved in the transfer of pleasure to desire, if only because belief is an effect of custom that rests on general rules, which themselves concern causality. But it is more particularly involved in the working of what Hume called "mixtures of passions", namely "hope" and "fear" where joy or grief are respectively dominant. These mixtures of passions lead to an analysis of choice under uncertainty, of which several features are given in Book II, Part III, Section IX of the *Treatise*, devoted to direct passions (Hume 1739-40, II, pp. 439 *sqq*.), in the first section of the *Dissertation* on the same subject (Hume 1757, pp. 139 *sqq*.), and in the numerous passages of both the *Treatise* and the first *Enquiry* which concern probability (specially, Hume 1739-40, I, pp. 124 *sqq*.; Hume 1748, pp. 56-9). The resulting desire appears as a transformation of the pleasure and pain involved in the mixed passions.

(2). The effect of *contiguity*, either in space or in time, is approached through several passages, in Book II of the *Treatise* (Hume 1739-40, II, pp. 418 *sqq*, 427 *sqq*, 432 *sqq*), in the second *Enquiry* (Hume 1751, pp. 239-40), and in the *Dissertation* (Hume 1757, pp. 161-2). This provides a key to Hume's approach to both intertemporal and spatial decision, like in Lapidus 2010, where it is shown that the belief in the idea of the pleasure

of a future good (the desire for this good) is negatively linked to the degree of violence of the passion, on which depends the action of the natural relation of contiguity.

(3). A remarkable effect of *resemblance* on the way pleasure is transferred to desire and volition is presented at the end of a discussion on probability in book I of the *Treatise* (Hume 1739-40, I, pp. 141-2). The procedure that is involved suggests a solution to the already mentioned problem of indiscrimination between impressions of pleasure.

The principle which governs these three topics remains the same: the strength of an idea (the "belief"), related to an impression of pleasure or pain varies when transferred onto desire and volition. The different origins of this variation (respectively: causality, resemblance or contiguity) correspond to separate fields in contemporary theory of decision (decision under uncertainty, indiscrimination, intertemporal and spatial decision) which thus appear, from a Humean point of view, closer to each other than it is usually acknowledged. From now on, it is obvious that "belief" should be understood in what we already called a *technical sense*, nonetheless different from the one which is employed in standard decision theory. We would hardly say, in usual language, that the resolution of a problem of indiscrimination, which involves resemblance, produces a belief; and in the same way, in regards to choice theory, it seems difficult to say that we believe less in a future good when no uncertainty occurs. However, all these cases refer to beliefs, in a Humean sense.

The juncture between the natural relation centrally involved in the formation of belief (1), (2) or (3) and the corresponding topic in decision theory is summarised below.

<table>
<tr><th colspan="2">Natural relations</th><th>Topics and sources</th></tr>
<tr><td colspan="2" rowspan="2">(1) Causality</td><td>Decision under risk or uncertainty
(Hume 1739-40, II, pp. 439 sqq; 1757, pp. 139 sqq.)</td></tr>
<tr><td rowspan="2">(2) Contiguity</td><td rowspan="2">Intertemporal and spatial decision
(Hume 1739-40, II, pp. 418-22, 427-38; 1751, pp. 239-40; 1757, pp. 161-2)</td></tr>
<tr><td rowspan="2"></td></tr>
<tr><td>(3) Resemblance</td><td>Indiscrimination
(Hume 1739-40, I, pp. 141-2)</td></tr>
</table>

The origins of belief in the formation of decision

This suggests that for each topic, belief might formally operate in a similar way in order to transform pleasure into desire. Whatever the natural relation mainly involved, the original pleasure relation $L_S(v)$ is, let us say, scrambled by belief in order to give birth to $R_S(v)$. In the case of the relation of contiguity, for instance, belief possibly changes the order

between such elements *x* and *y*, when moving from $L_S(v)$ to $R_S(v)$. Hume gives some general features of the procedure involved, which favours several kinds of dependencies of the relative places of two desirable objects, through contiguity itself, violence of the passion, memory, date, or target effects. Similar instances might be provided by causality alone, or resemblance.

Generally speaking, this procedure gives rise to a non-empty class of desire relations, shaped by beliefs resting on causality, contiguity, or resemblance, among which at least one is transitive. Although it is not self-evident, these desire relations $R_S(v)$, which appear as the outcome of the belief on the pleasure relation in a context S, are linked to the ones which were used to denote contextual preferences R_S in Diaye and Lapidus 2005a (the precise characteristics of the emotional state, given by *v*, was omitted), and whose completeness on S was established (Diaye and Lapidus 2005a, pp. 96-97). In this last paper, it was only assumed that contextual preferences R_S, which were not necessarily transitive at each step of a decision process denoted D-PROC, were changing and were revised all along this process, till the moment when they became stabilised and, as a result, transitive (*Ibid*., proposition 2, p. 106). The link between the desire relations $R_S(v)$ which stem from pleasure and belief, and the contextual preferences in D-PROC might be expressed through the following condition:

Density condition

> The set of contexts of choice faced by the individual is dense with respect to D-PROC, that is for any $R_S(v)$ there exists a sequence of revision of preferences in D-PROC of which it is an outcome [3.1]

Under this density condition, it is clear that preferences remain, at each step, shaped by belief on the basis of the relation of pleasure. Indeed, for any preference relation $R_S(v)$, the density condition implies that there always exists a sequence of contextual sets S_0, S_1, …S_i, … such that applying D-PROC over this sequence leads to a general preference R over $S_0 \cup S_1 \cup \ldots \cup S_i \cup ..$, whose restriction over S coincides with $R_S(v)$.

In the following subsection, this quite general procedure is illustrated by the working of the natural relation of resemblance in order to solve indiscrimination problems.

3.3. From Pleasure to Desire: Solving an Indiscrimination Problem

When dealing with what he called "minute differences" between amounts of money, Hume opened the path to an apparently simple way to solve the associate problem of indiscrimination between close impressions, using the natural relation of resemblance. Hume's procedure is introduced as follows:

> "The difference, therefore, of our conduct in preferring the greater number depends not upon our passions, but upon custom and *general rules.* We have found in a multitude of instances that the

> augmenting the numbers of any sum augments the passion, where the numbers are precise and the difference sensible. The mind can perceive, from its immediate feeling, that three guineas produce a greater passion than two; and *this* it transfers to larger numbers, because of the resemblance; and by a general rule assigns to a thousand guineas a stronger passion than to nine hundred and ninety-nine." (Hume 1739-40, I: 141-142).

It is assumed in this example that we are facing a simple context of choice S = {2; 3; 999; 1000} included in $\mathbb{R}_+$. Considering v as given, the correspondence of pleasure is such that

$\pi(1000, v) \cap \pi(999, v) \neq \varnothing$

$\pi(999, v) > \pi(3, v) > \pi(2, v)$,

and $\pi(1000, v) > \pi(3, v) > \pi(2, v)$.

In terms of the underlying pleasure relation $L_S(v)$, this means that

$(999, 3), (999, 2), (1000, 3), (1000, 2), (3, 2) \in PL_S(v)$

whereas $(999, 1000) \in IL_S(v)$.

Based on the resemblance of the differences between 3 and 2 guineas, and between 1000 and 999 guineas, Hume's procedure leads to believe more in the idea of the pleasure associated with 1000 than in the one associated with 999 guineas, in spite of the fact that our mind cannot discriminate between the correlate impressions of pleasure. This amounts to transform the pleasure relation $L_S(v)$ into a complete order of desire $R_S(v)$ as follows, ($PR_S(v)$ denoting the asymmetric part of $R_S(v)$) :

$(999, 3), (999, 2), (1000, 3), (1000, 2), (3, 2), (1000, 999) \in PR_S(v)$

And, of course, $R_S(v)$ can be represented by a scalar function $u(x, v)$:

$u(1000, v) > u(999, v) > u(3, v) > u(2, v)$

In the elementary case of a single good that, like an amount of money, varies in quantities, Hume's procedure makes it possible to cancel out the consequences of indiscrimination by transforming the quasi-transitive pleasure relation into a class of desire relations which contains a single element, which is a preorder of desire. However, the principle on which this simple procedure rests is more general. It amounts to construct desire relations $R_S(v)$ (subsets of $\mathbb{R}^n_+ \times \mathbb{R}^n_+$) on the basis of the pleasure relation $L_S(v)$ by adding or subtracting some structures, and to show that at least one of these relations is a preorder.

This more general resemblance-belief procedure is described by [3.2] and [3.3]:

- $PL_S(v) \subseteq PR_S(v)$ [3.2]
  (that is, the asymmetric part of the desire relation includes the asymmetric part of the pleasure relation).
- $\forall (x, y) \in IL_S(v)$,
  if $\exists (z, t) \in S^2$ with $(z, t) \in PL_S(v)$ and $x\text{-}y \geq z\text{-}t > 0$, then $(x, y) \in PR_S(v)$ [3.3]

(this extends Hume's example to the case where, for instance, S = {1, 2, 10000, 10001.1}).

Allowing [3.1], [3.2], and [3.3], the proposition and the corollary hereafter can be established:

Proposition

Let $L_S(v)$ be a complete quasi-transitive pleasure relation on S, which is represented by an underlying correspondence of pleasure $\pi_S(x, v) = [a_S(x, v), b_S(x, v)]$ (see [2.11] above), so that for all $x, y \in S$, $x\ IL_S(v)\ y \Rightarrow \pi_S(x, v) \cap \pi_S(y, v) \neq \varnothing$, and $x\ PL_S(v)\ y \Rightarrow \pi_S(x, v) > \pi_S(y, v)$ ([2.12] and [2.13] above), the monotony condition [2.14] being satisfied. The class of desire relations $R_S(v)$ derived from a pleasure relation $L_S(v)$ and fulfilling conditions [3.2] and [3.3] is not empty. Moreover it includes at least one preorder.

Proof : Because of [2.14], it is possible to construct at least one $R_S(v) = PR_S(v) \cup IR_S(v)$ in the following way:

$\forall x, y \in S$, $x\ PR_S(v)\ y$ only if $a_S(x, v) > a_S(y, v)$

$\forall x, y \in S$, $x\ IR_S(v)\ y$ only if $a_S(x, v) = a_S(y, v)$

Let us show that the resulting desire relation $R_S(v)$ satisfies [3.2] and [3.3].

- Let $x, y \in S$. $x\ PL_S(v)\ y \Rightarrow \pi_S(x, v) = [a_S(x, v), b_S(x, v)] > \pi_S(y, v) = [a_S(y, v), b_S(y, v)]$. As a consequence, $a_S(x, v) > a_S(y, v)$. Hence by construction of $R_S(v)$, we have $x\ PR_S(v)\ y$. This shows that [3.2] is satisfied.
- Let $(x, y) \in IL_S(v)$. Suppose that $\exists (z, t) \in S^2$ with $z\ PL_S(v)\ t$ and $x\text{-}y \geq z\text{-}t > 0$, then $x > y$. By monotony condition [2.14], we have $a_S(x, v) > a_S(y, v)$ and $b_S(x, v) > b_S(y, v)$. And by construction of $R_S(v)$, we have $(x, y) \in PR_S(v)$. This shows that [3.3] is satisfied.

Let us now show that this desire relation $R_S(v)$ is a preorder.

- It is obviously complete because it is constructed from $L_S(v)$ which is complete.
- It is also transitive. To see this, let us remark that the symmetric part of $R_S(v)$, namely $IR_S(v)$ is clearly transitive because for all $x, y \in S$, $x\ IR_S(v)\ y$ only if $a_S(x, v) = a_S(y, v)$. The asymmetric part of $R_S(v)$, namely $PR_S(v)$ is also transitive because for all $x, y \in S$, $x\ PR_S(v)\ y$ only if $a_S(x, v) > a_S(y, v)$.

■

Corollary

Suppose that the conditions stated in the above proposition and density condition [3.1] are satisfied. There exists a preorder $R_S(v)$ that belongs to the class of desire relations transformed by belief from the pleasure relation, and which constitutes an issue of D-PROC.

Proof : According to the above proposition, there exists a preorder $R_S(v)$ that belongs to the class of desire relations transformed by belief from the pleasure relation. However according to condition 3.1], there exists a sequence of revision of preferences in D-PROC of which $R_S(v)$ is an outcome.

■

Both the proposition and the corollary above express the embedment between the operation of belief that transforms pleasure into desire, and the decision process that possibly gives rise to the rationality of the latter and of the associated choice.

4. CONCLUSION

Hume's approach opens the path to a serious challenge to our current understanding of the decision process. The reason is *not* that standard categories and topics suddenly become meaningless: rationality, functional representation of preferences, on the one hand, or intertemporal decision, choice indiscrimination, decision under risk or uncertainty, on the other hand, all remain relevant. But they are not relevant in the same way.

Rationality, in the technical meaning of rationality of choice and preferences, gives birth to a more qualified approach. It is now considered not as a prerequisite, directly derived from a given set of axioms, but as a possible and interpretable outcome of a decision process. Moreover, the usual question of the functional representation of preferences is inverted. Instead of a convenient representation through a function of utility, which does not mean anything else than the underlying preorder of preferences, Hume's work puts forward an alternative approach where pleasure, considered as the value of a pleasure function or correspondence, expresses and transforms itself into a desire relation that displays the usual characteristics of a preference relation. As a result, desire has content in terms of pleasure, even if this content is not monotonously reflected in desire, whereas in standard analysis, utility represents preferences without possessing any specific content.

But such an operation is possible only because we accept the idea of some general device, which connects our pleasures and pains to our desires and aversions. This is precisely the role that is granted, from a Humean point of view, to belief. Through what Hume called the "natural relations" of the mind, belief unifies topics that we are used to considering separate. The belief that we add to the idea of a pleasure associated with an object distant in space or time, to the achievement of a risky event, or to an object so close to another that the discrimination power of our senses does not allow us to distinguish between them, performs in all these cases a similar operation: drawing on a structure of pleasure, it transforms this

latter into a structure of desire. As a result, Hume's challenge meets a recurrent ulterior motive in decision theory: providing a missing unifying principle.

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ABSTRACT

The purpose of this paper is to introduce explicitly pleasure and belief in what aims at being a Humean theory of decision, like the one developed in Diaye and Lapidus (2005a). Although we support the idea that Hume was in some way a hedonist – evidently different from Bentham's or Jevons' way – we lay emphasis less on continuity than on the specific kind of hedonism encountered in Hume's writings (chiefly the *Treatise*, the second *Enquiry*, the *Dissertation*, or some of his *Essays*). Such hedonism clearly contrasts to its standard modern inheritance, expressed by the relation between preferences and utility.

The reason for such a difference with the usual approach lies in the mental process that Hume puts to the fore in order to explain the way pleasure determines desires and volition. Whereas pleasure is primarily, in Hume's words, an *impression* of sensation, it takes place in the birth of passions as reflecting an *idea* of pleasure, whose "force and vivacity" is precisely a "belief", transferred to the direct passions of desire or volition that come immediately before action. As a result, from a Humean point of view, "belief" deals with decision under risk or uncertainty, as well with intertemporal decision and indiscrimination problems.

The latter are explored within a formal framework, and it is shown that the relation of pleasure is transformed by belief into a non-empty class of relations of desire, among which at least one is a preorder.