Risk Aversion in the Euro area

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Abstract

We propose a New Keynesian Dynamic Stochastic General Equilibrium (DSGE) model where a risk aversion shock enters a separable utility function. We analyze five periods, each one lasting twenty years, to follow over time the dynamics of several parameters (such as the risk aversion parameter), the Taylor rule coefficients and the role of this risk aversion shock on output and real money balances in the Eurozone. Our analysis suggests that risk aversion was a more important component of output and real money balance dynamics between 2006 and 2011 than it had been between 1971 and 2006, at least in the short run.

Keywords: Risk aversion, Output, Money, Euro area, New Keynesian DSGE models, Bayesian estimation.

JEL Classification Number: E23, E31, E51.

1 Introduction

The New Keynesian model, as developed by Galí (2008) and Walsh (2010), brings three equations together to characterize the dynamic behavior of three macroeconomic key variables: output, inflation, and the nominal interest rate. The resulting output equation corresponds to the log-linearization of an optimizing household's Euler equation, linking consumption and output growth to the inflation-adjusted return on nominal bonds, that is, to the real interest rate. The inflation equation describes the optimizing behavior of monopolistically competitive firms that either set prices in a randomly staggered fashion, as suggested by Calvo (1983), or face explicit costs of nominal price adjustment, as suggested by Rotemberg (1982). The nominal interest rate equation, a monetary policy rule of the kind proposed by Taylor (1993), dictates that the central bank should adjust the short-term nominal interest rate in response to a trade-off between changes in inflation and output and changes in the past interest rate.

In this framework, even if money or real balances are included in the utility (MIU) or in the central bank's reaction function, real or nominal monetary aggregates generally become an irrelevant variable and are then neglected, at least for the US, as in Woodford (2003) and Ireland (2004). Additionally, for the Eurozone, even if a money variable appears in the monetary policy reaction function, as in Andrés, López-Salido and Vallés (2006) or in Barthélemy, Clerc, and Marx (2011), money plays no role in the dynamics.

However, Benchimol and Fourçans (2012) show that the role of money in the business cycle is dependent on the risk aversion level, at least in the Eurozone. They estimate a New Keynesian DSGE model with non-separable household preferences between consumption and money, as in the Ireland (2004) or Andrés, López-Salido, and Nelson (2009) to analyze the role of money in the dynamics of the variables under a high level of risk aversion. In this context, they establish a significant link between money, output and risk by showing that real money has a significant role with regard to output and flexible-price output dynamics in the short term only if the relative risk aversion level is sufficiently high (twice the standard value). They study the role of the level of risk aversion in a non-standard MIU function case and do not include a standard case or a study of a micro-founded risk aversion shock for the Eurozone. They only consider standard micro-founded shocks (price-markup, monetary policy and technology) and a money demand shock.

Finally, as in other studies, we also consider a price-markup shock, a monetary policy shock and a technology shock. To analyze the role of risk aversion in the dynamics of other variables, we do not consider a money shock that has no role in the dynamics of this framework (because of the separa-

bility assumption between consumption and money), as shown by Smets and Wouters (2003), but we do consider a money equation to take account of the behaviors of national central banks (before 1999) and the European Central Bank (after 1999) and to close the model as much with historical variables as with exogenous shocks.

In contrast, as relative risk aversion measures the willingness to substitute consumption over different periods, the lower the level of risk aversion, the more households substitute consumption over time. Wachter (2006) and Bekaert, Engstrom, and Grenadier (2010) show that an increase in risk aversion involve an increase in equity and bond premiums and may increase the real interest rate through a consumption smoothing effect or decrease it through a precautionary savings effect. Bommier, Chassagnon, and Le Grand (2012) also show that risk aversion enhances precautionary savings. These studies confirm the potential link between money holdings, output and risk aversion.

However, few studies quantify this link, or even consider risk aversion as a shock, in a New Keynesian DSGE framework. Moreover, no studies use Bayesian techniques as Fernández-Villaverde (2010) does to analyze the role of the risk aversion shock in output and money dynamics in the Eurozone. In the nearly same technical and theoretical context, Alpanda (2012) highlights the important role played by risk aversion shocks in US output between 2006 and 2011.

Accordingly, this article contributes to the literature in several ways. First, we analyze the role of a micro-founded risk aversion shock in the dynamics of a New Keynesian DSGE model. Second, the development of a completely micro-founded model with a risk aversion shock is original in terms of findings as well as in terms of estimation techniques. Mainly inspired by Smets and Wouters (2007) and Galí (2008), our model explores the role of risk aversion in inflation, output, interest rate and real money balances, as well as in flexible-price output..

A specific emphasis will be placed on how the risk aversion shock impacts the dynamics of these key variables through time. We use Bayesian techniques, as in An and Schorfheide (2007), to estimate five subsamples of the Eurozone between 1971 and 2011, each one lasting twenty years. This original focus on the last forty years will show that risk aversion shocks have had stronger effects on output and real money balances in recent years than in the more distant past.

Last but not least, our framework allows us to analyze successively the informational content of the last two crises (subprimes and sovereign debts) in comparison with other crises that occurred between 1971 and 2006 in the Euro area.

Bayesian estimations and dynamic analyses of the model, with impulse response functions and short- and long-run variance decompositions following structural shocks, yield different relationships between risk aversion and other structural variables. This approach sheds light on the importance of risk aversion and its impact on output and real money balances during the last five years (2006 to 2011). It also shows that the role of monetary policy as regards output in the short run has decreased in the recent years in comparison to the more distant past.

Finally, this study explores with modern theoretical and empirical tools a fundamental question about the role of the perception of economic risks, e.g., the ability for households to consume now or later, in the dynamics of the main economic variables for the Eurozone.

Section 2 describes the theoretical set up. In Section 3, the model is calibrated and estimated with Euro area data and impulse response functions and variance decomposition are analyzed. Interpretation of the results is provided in Section 4. Section 5 concludes, and the Appendix presents additional theoretical and empirical results.

2 The model

The model consists of households that supply labor, purchase goods for consumption and hold money and bonds, and of firms that hire labor and produce and sell differentiated products in monopolistically competitive goods markets. Each firm sets the price of the good it produces, but not all firms reset their price during each period. Households and firms behave optimally: households maximize the expected present value of utility, and firms maximize profits. There is also a central bank that controls the nominal rate of interest. This model is essentially inspired by Smets and Wouters (2007) and Galí (2008).

2.1 Households

We assume a representative infinitely lived household, seeking to maximize

$$E_t \left[\sum_{k=0}^{\infty} \beta^k U_{t+k} \right] \tag{1}$$

where U_t is the period utility function and $\beta < 1$ is the discount factor. The household decides how to allocate its consumption expenditures among the different goods. This requires that the consumption index C_t be maximized

for any given level of expenditures, as in Galí (2008). Furthermore, and conditional on such optimal behavior, the period budget constraint takes the form

$$P_t C_t + M_t + Q_t B_t \leq B_{t-1} + W_t N_t + M_{t-1} \qquad (2)$$

where $t = 0, 1, 2...$, P_t is an aggregate price index, M_t is the quantity of money holdings at time t, B_t is the quantity of one-period nominally riskless discount bonds purchased in period t and maturing in period $t+1$ (each bond pays one unit of money at maturity and its price is Q_t where $i_t = -\log Q_t$ is the short term nominal rate), W_t is the nominal wage, and N_t is hours of work (or the measure of household members employed). The above sequence of period budget constraints is supplemented with a solvency condition[1].

Preferences are measured with a common time-separable utility function (MIU). Under the assumption of a period utility given by

$$U_t = \frac{C_t^{1-\sigma_t}}{1 - \sigma_t} + \frac{\gamma}{1 - \nu} \left(\frac{M_t}{P_t} \right)^{1-\nu} - \frac{\chi N_t^{1+\eta}}{1 + \eta} \qquad (3)$$

consumption, money demand, labor supply, and bond holdings are chosen to maximize (1) subject to (2) and the solvency condition. This MIU utility function depends positively on the consumption of goods, C_t, positively on real money balances, $\frac{M_t}{P_t}$, and negatively on labor N_t. $\sigma_t = \sigma + \varepsilon_t^r$ is the time-varying coefficient of the relative risk aversion of households (or the inverse of the intertemporal elasticity of substitution), where ε_t^r is a risk aversion shock. ν is the inverse of the elasticity of money holdings with respect to the interest rate, and η is the inverse of the elasticity of work effort with respect to the real wage. γ and χ are positive scale parameters.

This setting leads to the following conditions[2], which, in addition to the budget constraint, must hold in equilibrium. The resulting log-linear version of the first-order condition corresponding to the demand for contingent bonds implies that

$$c_t = E_t [c_{t+1}] - \frac{1}{\sigma_t} (i_t - E_t [\pi_{t+1}] - \rho_c) \qquad (4)$$

where $c_t = \ln (C_t)$ is the logarithm of the aggregate consumption, i_t is the nominal interest rate, $E_t [\pi_{t+1}]$ is the expected inflation rate in period $t + 1$ with knowledge of the information in period t, and $\rho_c = -\ln (\beta)$.

The demand for cash that follows from the household's optimization problem is given by

$$\sigma_t c_t - \nu m p_t - \rho_m = a_2 i_t \qquad (5)$$

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[1] Such as $\forall t \lim_{n \longrightarrow \infty} E_t [B_n] \geq 0$, in order to avoid Ponzi-type schemes.

[2] See Appendix 6.A

where $mp_t = m_t - p_t$ are the log linearized real money balances, $\rho_m = -\ln(\gamma) + a_1$, and a_1 and a_2 are resulting terms of the first-order Taylor approximation of $\log(1 - Q_t) = a_1 + a_2 i_t$.

Real cash holdings depend positively on consumption with an elasticity equal to $\frac{\sigma_t}{\nu}$ and negatively on the nominal interest rate[3]. In what follows, we take the nominal interest rate as the central bank's policy instrument.

The resulting log-linear version of the first-order condition corresponding to the optimal consumption-leisure arbitrage implies that

$$w_t - p_t = \sigma_t c_t + \eta n_t - \rho_n \tag{6}$$

where $w_t - p_t$ corresponds to the log of the real wage, n_t denotes the log of hours of work, and $\rho_n = -\ln(\chi)$.

Finally, these equations represent the Euler condition for the optimal intratemporal allocation of consumption (Eq. (4)), the intertemporal optimality condition setting the marginal rate of substitution between money and consumption equal to the opportunity cost of holding money (Eq. (5)), and the intratemporal optimality condition setting the marginal rate of substitution between leisure and consumption equal to the real wage (Eq. (6)).

2.2 Firms

Backus, Kehoe, and Kydland (1992) have shown that capital appears to play a rather minor role in the business cycle. To simplify the analysis and focus on the role of risk, we do not include a capital accumulation process in this model, as in Galí (2008).

We assume a continuum of firms indexed by $i \in [0, 1]$. Each firm produces a differentiated good, but they all use an identical technology, represented by the following production function

$$Y_t(i) = A_t N_t(i)^{1-\alpha} \tag{7}$$

where $A_t = \exp(\varepsilon_t^a)$ represents the level of technology, assumed to be common to all firms and to evolve exogenously over time, and ε_t^a is a technology shock.

All firms face an identical isoelastic demand schedule and take the aggregate price level P_t and aggregate consumption index C_t as given. As in the standard Calvo (1983) model, our generalization features monopolistic competition and staggered price setting. At any time t, only a fraction $1 - \theta$ of firms, with $0 < \theta < 1$, can reset their prices optimally, while the remaining firms index their prices to lagged inflation.

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[3]Because $\frac{1}{\beta} > 1$, $a_2 > 0$.

2.3 Central bank

The central bank is assumed to set its nominal interest rate according to an augmented smoothed Taylor (1993) rule such as:

$$i_t = (1 - \lambda_i) \left(\lambda_\pi (\pi_t - \pi_c) + \lambda_x \left(y_t - y_t^f \right) + \lambda_m (mp_t - mp_c) \right) + \lambda_i i_{t-1} + \varepsilon_t^i \tag{8}$$

where λ_π, λ_x and λ_m are policy coefficients reflecting, respectively, the weight on inflation, the output gap and real money; the parameter $0 < \lambda_i < 1$ captures the degree of interest rate smoothing; and ε_t^i is an exogenous *ad hoc* shock accounting for fluctuations of the nominal interest rate. π_c is an inflation target, and mp_c is a money target, essentially included to account for changes in targeting policies of inflation and monetary aggregates, as in, respectively, Svensson (1999) and Fourçans and Vranceanu (2007)[4].

λ_m takes also into account the potential national central bank's money targeting before the creation of the European Central Bank (ECB, 1999). After 1999, the ECB follows an explicit money targeting until 2004 called the *Two Pillars* policy, as explained in Barthélemy, Clerc, and Marx (2011), and may even follow an implicit one after this date, as suggested by Kahn and Benolkin (2007).

3 Empirical results

3.1 DSGE model

Our macro model consists of five equations and five dependent variables: inflation, nominal interest rate, output, real money balances, and flexible-price output. Flexible-price output is completely determined by shocks.

$$y_t^f = \frac{1 + \eta}{\sigma_t (1 - \alpha) + \eta + \alpha} \varepsilon_t^a + \frac{(1 - \alpha) \left(\log (1 - \alpha) + \rho_n - \log \left(\frac{\varepsilon}{\varepsilon - 1} \right) \right)}{\sigma_t (1 - \alpha) + \eta + \alpha} \tag{9}$$

$$\pi_t = \beta E_t [\pi_{t+1}] + \frac{(1 - \theta)(1 - \beta\theta)(\sigma_t (1 - \alpha) + \eta + \alpha)}{\theta (1 - \alpha + \alpha\varepsilon)} \left(y_t - y_t^f \right) \tag{10}$$

$$y_t = E_t [y_{t+1}] - \sigma_t^{-1} (i_t - E_t [\pi_{t+1}] - \rho_c) \tag{11}$$

$$mp_t = \frac{\sigma_t}{\nu} y_t - \frac{a_2}{\nu} i_t - \frac{\rho_m}{\nu} \tag{12}$$

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[4]Other studies introduce a relevant money variable in the Euro area Taylor rule: Andrés, López-Salido and Vallés (2006), Andrés, López-Salido, and Nelson (2009), Barthélemy, Clerc, and Marx (2011), Benchimol and Fourçans (2012).

$$i_t = (1 - \lambda_i) \left(\lambda_\pi (\pi_t - \pi_c) + \lambda_x \left(y_t - y_t^f \right) + \lambda_m (mp_t - mp_c) \right) + \lambda_i i_{t-1} + \varepsilon_t^i$$

$$(13)$$

where $a_1 = \log \left(1 - e^{-\frac{1}{\beta}} \right) - \frac{\frac{1}{\beta}}{e^{\frac{1}{\beta}} - 1}$ and $a_2 = \frac{1}{e^{\frac{1}{\beta}} - 1}$.

All structural shocks are assumed to follow a first-order autoregressive process with an *i.i.d.* normal error term, such as $\varepsilon_t^k = \mu_k \varepsilon_{t-1}^k + \omega_{k,t}$, where $\varepsilon_{k,t} \sim N(0; \sigma_k)$ for $k = \{p, i, a, r\}$.

3.2 Euro area data

In our model of the Eurozone, π_t is the detrended inflation rate measured as the yearly log difference of the detrended GDP Deflator from one quarter to the same quarter of the previous year; y_t is the detrended output per capita measured as the difference between the log of the real GDP per capita and its trend; and i_t is the short-term (3-month) detrended nominal interest rate. These data are extracted from the AWM database of Fagan, Henry, and Mestre (2001). mp_t is the detrended real money balances per capita measured as the difference between the real money per capita and its trend, where real money per capita is measured as the log difference between the money stock per capita and the GDP Deflator. We use the $M3$ monetary aggregate from the Eurostat database.

3.3 Calibration

Following standard conventions, we calibrate beta distributions for parameters that fall between zero and one, inverted gamma distributions for parameters that need to be constrained to be greater than zero, and normal distributions in other cases.

The parameters of the utility function are assumed to be distributed as follows. Only the discount factor is fixed in the estimation procedure to 0.98. The intertemporal elasticity of substitution (*i.e.* the level of relative risk aversion) is set at 2, a mean between the calibrations of Rabanal and Rubio-Ramírez (2005) and Casares (2007), and consistent with the calibrated value used by Kollmann (2001) and the value estimated by Lindé, Nessén, and Söderström (2009). The inverse of the Frisch elasticity of labor supply is assumed to be approximately 1, as in Galí (2008) and the scale parameters on money and labor are assumed to be approximately 0.2, as in Benchimol and Fourçans (2012).

The calibration of α, θ, and ε comes from Smets and Wouters (2007), Casares (2007) and Galí (2008). The smoothed Taylor rule (λ_i, λ_π, λ_x and

λ_m) priors are calibrated following Smets and Wouters (2003), Andrés, López-Salido, and Nelson (2009), and Barthélemy, Clerc, and Marx (2011). To observe both the behavior of the central bank and risk aversion, we assign a higher standard error (0.2) and a Normal prior law for the relative risk aversion level and for the Taylor rule's coefficients (including the inflation and money targets), except for the smoothing parameter, which is restricted to be positive and less than one (Beta distribution). The inflation target, π_c, is calibrated to 2%, and the money target, mp_c, is assumed to be approximately 4%.

The calibration of the shock persistence parameters and the standard errors of the innovations follow Smets and Wouters (2007). All of the standard errors of shocks are assumed to be distributed according to inverted Gamma distributions, with prior means of 0.01. The latter ensures that these parameters have positive support. The autoregressive parameters are all assumed to follow Beta distributions. All of these distributions are centered approximately 0.75, except for the autoregressive parameter of the monetary policy shock and the risk aversion shock, which are centered approximately 0.50, as in Smets and Wouters (2007). We take a common standard error of 0.15 for the shock persistence parameters, which is a mean between that of Benchimol and Fourçans (2012) and Smets and Wouters (2007).

	Law	Mean	Std.		Law	Mean	Std.
α	beta	0.33	0.10	λ_m	normal	1.00	0.20
θ	beta	0.66	0.10	π_c	normal	2.00	0.20
σ	normal	2.00	0.20	mp_c	normal	4.00	0.20
v	normal	1.50	0.10	ρ_a	beta	0.75	0.15
ε	normal	6.00	0.10	ρ_p	beta	0.75	0.15
η	normal	1.00	0.10	ρ_i	beta	0.50	0.15
γ	beta	0.20	0.05	ρ_r	beta	0.50	0.15
χ	beta	0.20	0.05	σ_a	invgamma	0.01	2.00
λ_i	beta	0.50	0.10	σ_p	invgamma	0.01	2.00
λ_π	normal	3.00	0.20	σ_i	invgamma	0.01	2.00
λ_x	normal	1.50	0.20	σ_r	invgamma	0.01	2.00

Table 1: Priors summary

3.4 Results

The model is estimated with 160 observations from 1971 (Q1) to 2011 (Q1) with Bayesian techniques, as in Smets and Wouters (2007). However, to capture different policies and risk perceptions in the Euro area between 1971 and 2011, and more specifically between 2006 and 2011, we divide this large sample into five subsamples, each one consisting of 80 observations (20 years).

This procedure allows us to analyze five different periods with a sufficiently large sample, as specified in Fernandez-Villaverde and Rubio-Ramirez (2004). Accordingly, we estimate our model over five different periods: from 1971Q1 to 1991Q1 (P1); from 1976Q1 to 1996Q1 (P2); from 1981Q1 to 2001Q1 (P3); from 1986Q1 to 2006Q1 (P4); and from 1991Q1 to 2011Q1 (P5).
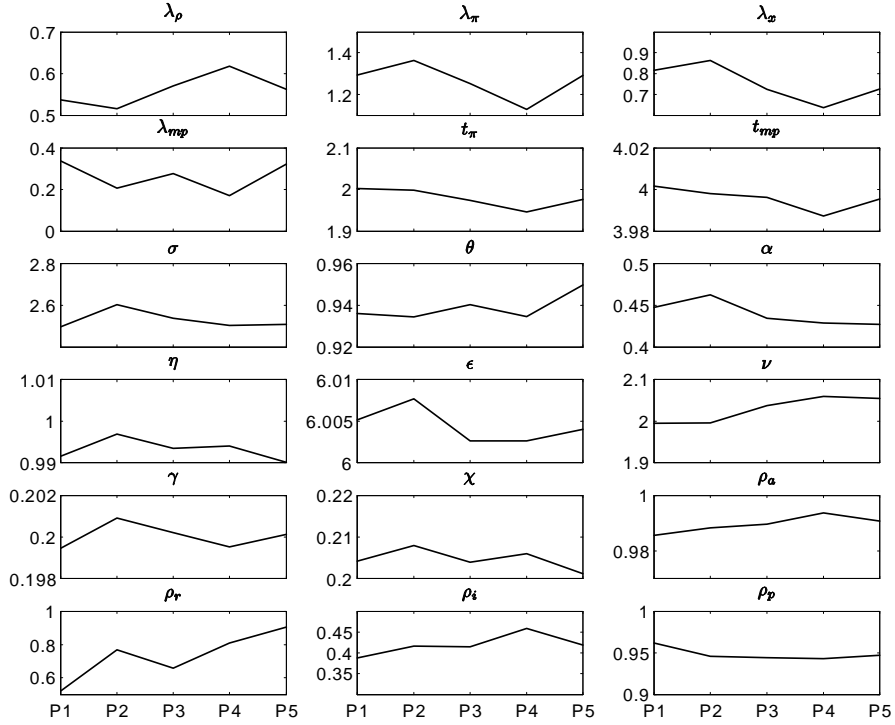

Figure 1: Bayesian estimation of parameters over the selected periods

The estimation of the implied posterior distribution of the parameters over the five periods (Fig. 1) is performed using the Metropolis-Hastings algorithm (10 distinct chains, each of 100000 draws). The average acceptance rates per chain are included in the interval $[0.19; 0.22]$ and the student's t-tests are all above 1.96. To assess the model validation, we insure convergence of the proposed distribution to the target distribution for each period in

Appendix 6.B. Priors and posteriors distributions are presented in Appendix 6.C.

3.5 Simulations

3.5.1 Impulse response functions

As in the literature, Appendix 6.D (Fig. 9) shows that a price-markup shock increases inflation and the nominal interest rate and decreases output, the output gap, the real interest rate, real money balances and real money growth.

The response of output, real money balances and real money growth to a technology shock is positive (Fig. 9). Notice that the improvement in technology is partly accommodated by the central bank, which lowers the nominal and real interest rate, while increasing the quantity of money in circulation.

Fig. 9 also presents the response to an interest rate shock. Inflation, output and the output gap, real money balances and real money growth all fall. The real and nominal interest rate rise.

This case is very interesting because Fig. 9 shows that a risk aversion shock leads to a decrease in output and an increase in inflation: it implies a tightening of monetary policy (because of the strong weight that the central banker places on inflation), and its strength depends on the period (strong monetary policy tightening in P1 and low monetary policy tightening in P5). The risk aversion shock also implies an increase in real money balances and real money growth and a decrease in the output gap.

Household consumption is reduced (decreasing output), and companies increase their price (to face high risk aversion and possibly low consumption), which implies an increase in the inflation rate, constrained by a tightening of monetary policy.

3.5.2 Variance decompositions

We analyze the forecast error variance decomposition of each variable following exogenous shocks. The analysis is conducted via an unconditional variance decomposition to analyze long-term variance decomposition (the grey bar in Fig. 2) and via a conditional variance decomposition, conditionally to the first period, to analyze short-run variance decomposition (the black bar in Fig. 2).

Fig. 2 shows that output is mainly explained by the technology shock in the long run (approximately 90%) and by the monetary policy shock (ap-
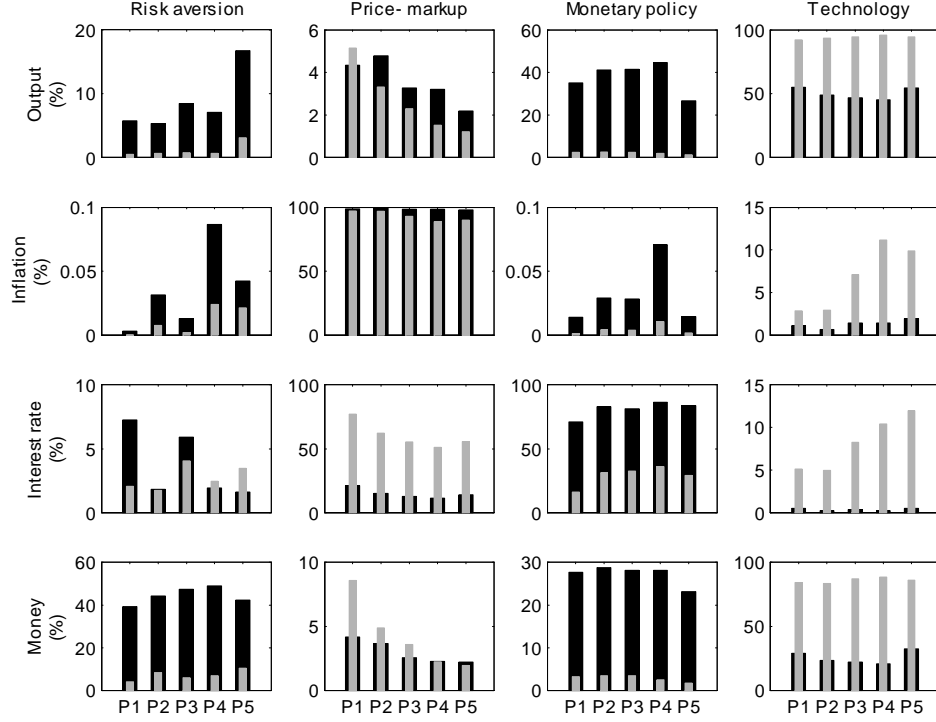
Figure 2: Long-term (grey) and short-term (black) variance decompositions

proximately 35%) and the technology shock (approximately 50%) in the short run. The rest of the variance in output is explained by the risk aversion shock (approximately 5% from P1 to P4 and more than 15% for P5) in the short term, whereas risk aversion shock has a limited role in output variance in the long run.

Fig. 2 also shows that, in accordance with the literature, inflation is mainly explained by the price-markup shock and that interest rate variance is mainly driven by monetary policy in the short run and by monetary policy and price-markup in the long run. Furthermore, most of the variance in real money balances is induced by the risk aversion shock (approximately 40%) and the monetary policy shock (approximately 25%) in the short run, whereas in the long run, real money balance variance is mainly driven by the technology shock. All of these results are in line with the literature.

4 Interpretation

Appendix 6.B shows that the estimation results are valid and that convergence is obtained for all estimations and all moments. Appendix 6.C shows

that the maximum of the posterior distribution reaches the posterior mean of each estimated parameter. The estimation is relatively well identified, and the data are quite informative for most of the estimated micro-parameters.

Fig. 9 shows that from P1 to P5, the impact of the price-markup shock on inflation and output is almost halved. It also shows that the risk aversion shock has a longer impact in P5 than it does in the other periods. This is due to the increase over the periods of the autoregressive parameter of the risk aversion shock, ρ_r, as shown in Fig. 1.

Fig. 2 shows that output and real money balances variances have an important component coming from the risk aversion shock. This finding shows the leading role of relative risk aversion in the dynamics of output, as in Black and Dowd (2011), and of real money balances, as in Benchimol and Fourçans (2012). Although the inflation rate, the nominal interest rate and the flexible-price output are strong components of output, risk aversion has a minor role to play in the variance of inflation and interest rate, and it has also no role to play with regard to the flexible-price output (less than 0.2% in the short and long run), which is completely determined by the technology shock. It also shows that inflation and interest rate variances are quasi unaffected by the introduction of the risk aversion shock, letting these variables be mainly explained by, respectively, the price-markup shock and the monetary policy shock.

The leading role of the risk aversion shock in the dynamics of real money balances in the short run is another important finding. Fig. 2 shows that real money balances are mainly explained by the technology shock (approximately 80%) in the long run, whereas in the short run, real money balances are mainly explained by the risk aversion shock and the monetary policy shock.

Fig. 2 shows that technology plays an increasingly important role in the short term for the inflation rate and, thus, the interest rate through the selected periods. This figure also shows that, in the short run, risk aversion has a more significant role in output dynamics in the last period (P5) than in the other periods (P1 to P4). This finding reflects the increasing role assumed by risk aversion in more recent years (between 2006 and 2011) as compared to the past (between 1971 and 2006).

Finally, Fig. 2 shows that monetary policy has a lower role in the short run concerning output in the last period (P5), approximately 22%, than it had in the past, approximately 35%. It highlights the transfer from the monetary policy role to the risk aversion role during the recent years. This confirms the declining influence of European monetary policy relative to the influence of risk aversion shocks.

5 Conclusion

Risk aversion is a concept in economics and finance that is based on the behavior of consumers and investors who are exposed to uncertainty. It is the reluctance of a person to accept a bargain with an uncertain payoff rather than another bargain offering a more certain, but possibly lower, expected payoff.

This paper presents a standard New Keynesian DSGE model that includes a risk aversion shock. It shows the involvement of this risk aversion shock in the dynamics of the economy: it increases inflation, decreases output (Fig. 9) and diminishes the impact of the central bank's actions on output variance, at least in the short run (Fig. 2). Risk aversion plays also an important role for output and real money balance dynamics. The negative role played by risk aversion in determining output is clearly identified, whereas it increases real money balances and real money growth in the first periods (Fig. 9).

Moreover, while estimations are quite robust (Fig. 3 to Fig. 8), they show that the risk aversion shock has a stronger impact on output dynamics during the last twenty years (P5) as compared to other analyzed periods (P1 to P4). This result is explained by the inclusion in P5 of the subprime and sovereign debt crises from 2007 to 2011.

This enhanced baseline model shows the importance of such a parameter to the economy, and especially its impact on output, money, and monetary policy. It also serves to show how it is important to control shocks to the risk aversion of agents, by communication for example.

6 Appendix

A Solving the model

- **Price dynamics**

Let's assume a set of firms not reoptimizing their posted prices in period t. Using the definition of the aggregate price level and the fact that all firms resetting prices choose an identical price P_t^*, leads to $P_t = \left[\theta P_{t-1}^{1-\Lambda_t} + (1-\theta)(P_t^*)^{1-\Lambda_t}\right]^{\frac{1}{1-\Lambda_t}}$, where $\Lambda_t = 1 + \frac{1}{\frac{1}{\varepsilon-1} + \varepsilon_t^p}$ is the elasticity of substitution between consumption goods in period t, and $\frac{\Lambda_t}{\Lambda_t - 1}$ is the markup of prices over marginal costs (time varying). Dividing both sides by P_{t-1} and log-linearizing around $P_t^* = P_{t-1}$ yields

$$\pi_t = (1-\theta)(p_t^* - p_{t-1}) \tag{14}$$

In this setup, we do not assume inertial dynamics of prices. Inflation results from the fact that firms reoptimizing in any given period their price plans, choose a price that differs from the economy's average price in the previous period.

- **Price setting**

A firm reoptimizing in period t chooses the price P_t^* that maximizes the current market value of the profits generated while that price remains effective. This problem is solved and leads to a first-order Taylor expansion around the zero inflation steady state:

$$p_t^* - p_{t-1} = (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t \left[\widehat{mc}_{t+k|t} + (p_{t+k} - p_{t-1}) \right] \qquad (15)$$

where $\widehat{mc}_{t+k|t} = mc_{t+k|t} - mc$ denotes the log deviation of marginal cost from its steady state value $mc = -\mu$, and $\mu = \log\left(\frac{\varepsilon}{\varepsilon-1}\right)$ is the log of the desired gross markup.

- **Equilibrium**

Market clearing in the goods market requires $Y_t(i) = C_t(i)$ for all $i \in [0, 1]$ and all t. Aggregate output is defined as $Y_t = \left(\int_0^1 Y_t(i)^{1-\frac{1}{\Lambda_t}} di \right)^{\frac{\Lambda_t}{\Lambda_t - 1}}$; it follows that $Y_t = C_t$ must hold for all t. One can combine the above goods market clearing condition with the consumer's Euler equation (4) to yield the equilibrium condition

$$y_t = E_t[y_{t+1}] - \sigma_t^{-1}(i_t - E_t[\pi_{t+1}] - \rho_c) \qquad (16)$$

Market clearing in the labor market requires $N_t = \int_0^1 N_t(i)\, di$. With the production function (7) and taking logs, one can write the following approximate relationship between aggregate output, employment and technology as

$$y_t = \varepsilon_t^a + (1 - \alpha) n_t \qquad (17)$$

An expression is derived for an individual firm's marginal cost in terms of the economy's average real marginal cost:

$$
\begin{aligned}
mc_t &= (w_t - p_t) - mpn_t \qquad &(18)\\
&= w_t - p_t - \frac{1}{1 - \alpha}(\varepsilon_t^a - \alpha y_t) - \log(1 - \alpha)
\end{aligned}
$$

for all t, where mpn_t defines the economy's average marginal product of labor. As $mc_{t+k|t} = (w_{t+k} - p_{t+k}) - mpn_{t+k|t}$ we have

$$mc_{t+k|t} = mc_{t+k} - \frac{\alpha \Lambda_t}{1 - \alpha} \left(p_t^* - p_{t+k} \right) \tag{19}$$

where the second equality follows from the demand schedule combined with the market clearing condition $c_t = y_t$. Substituting (19) into (15) yields

$$p_t^* - p_{t-1} = (1 - \beta\theta) \sum_{k=0}^{\infty} \Theta_{t+k} (\beta\theta)^k E_t \left[\widehat{mc}_{t+k} \right] + \sum_{k=0}^{\infty} (\beta\theta)^k E_t \left[\pi_{t+k} \right] \tag{20}$$

where $\Theta_t = \frac{1-\alpha}{1-\alpha+\alpha\Lambda_t} \leq 1$ is time varying to take into account the markup shock.

Finally, (14) and (20) yield the inflation equation

$$\pi_t = \beta E_t \left[\pi_{t+1} \right] + \lambda_{mc_t} \widehat{mc}_t \tag{21}$$

where β, $\lambda_{mc_t} = \frac{(1-\theta)(1-\beta\theta)}{\theta} \Theta_t$. λ_{mc_t} is strictly decreasing in the index of price stickiness θ, in the measure of decreasing returns α, and in the demand elasticity Λ_t.

Next, a relationship is derived between the economy's real marginal cost and a measure of aggregate economic activity. From (6) and (17), the average real marginal cost can be expressed as

$$mc_t = \left(\sigma_t + \frac{\eta + \alpha}{1 - \alpha} \right) y_t - \frac{1 + \eta}{1 - \alpha} \varepsilon_t^a - \log(1 - \alpha) - \rho_n \tag{22}$$

Under flexible prices, the real marginal cost is constant and equal to $mc = -\mu$. Defining the natural level of output, denoted by y_t^f, as the equilibrium level of output under flexible prices leads to

$$mc = \left(\sigma_t + \frac{\eta + \alpha}{1 - \alpha} \right) y_t^f - \frac{1 + \eta}{1 - \alpha} \varepsilon_t^a - \log(1 - \alpha) - \rho_n \tag{23}$$

thus implying

$$y_t^f = \upsilon_a \varepsilon_t^a + \upsilon_c \tag{24}$$

where $\upsilon_a = \frac{1+\eta}{\sigma_t(1-\alpha)+\eta+\alpha}$ and $\upsilon_c = \frac{(1-\alpha)\left(\log(1-\alpha)+\rho_n-\log\left(\frac{\varepsilon}{\varepsilon-1}\right) \right)}{\sigma_t(1-\alpha)+\eta+\alpha}$. Subtracting (25) from (24) yields

$$\widehat{mc}_t = \left(\sigma_t + \frac{\eta + \alpha}{1 - \alpha} \right) \left(y_t - y_t^f \right) \tag{25}$$

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where $\widehat{mc}_t = mc_t - mc$ is the *real marginal cost gap* and $y_t - y_t^f$ is the *output gap*. Combining the above equation with (23), we obtain

$$\pi_t = \beta E_t \left[\pi_{t+1}\right] + \psi_x \left(y_t - y_t^f\right) \tag{26}$$

where $\psi_x = \frac{(1-\theta)(1-\beta\theta)(\sigma_t(1-\alpha)+\eta+\alpha)}{\theta(1-\alpha+\alpha\varepsilon)}$ and $y_t - y_t^f$ is the *output gap*.

The second key equation describing the equilibrium of the model is obtained by rewriting (19) to determine output

$$y_t = E_t \left[y_{t+1}\right] - \sigma_t^{-1} \left(i_t - E_t \left[\pi_{t+1}\right] - \rho_c\right) \tag{27}$$

Equation (27) is thus a dynamic IS equation including the real money balances.

The third key equation describes the behavior of the *real money balances*. From (5), we obtain

$$mp_t = \frac{\sigma_t}{\nu} y_t - \frac{a_2}{\nu} i_t - \frac{\rho_m}{\nu} \tag{28}$$

B   Model validation

The red and blue lines in Fig. 3 represent an aggregate measure based on the eigenvalues of the variance-covariance matrix of each parameter both within and between chains. Each graph represents specific convergence measures and has two distinct lines that represent the results within and between chains. Those measures are related to the analysis of the parameter's mean (first moment), variance (second moment) and third moment of the model in the considered period. Convergence requires that both lines for each of the three measures become relatively constant and converge to each other.

The diagnoses concerning the numerical maximization of the posterior kernel indicate that the optimization procedure was able to obtain a robust maximum for the posterior kernel. A diagnosis of the overall convergence for the Metropolis-Hastings sampling algorithm is provided in Fig. 3.

Diagnoses for each individual parameter were also obtained, following the same structure as that of the overall. Most of the parameters do not seem to exhibit convergence problems, notwithstanding the fact that this evidence is stronger for some parameters than it is for others.
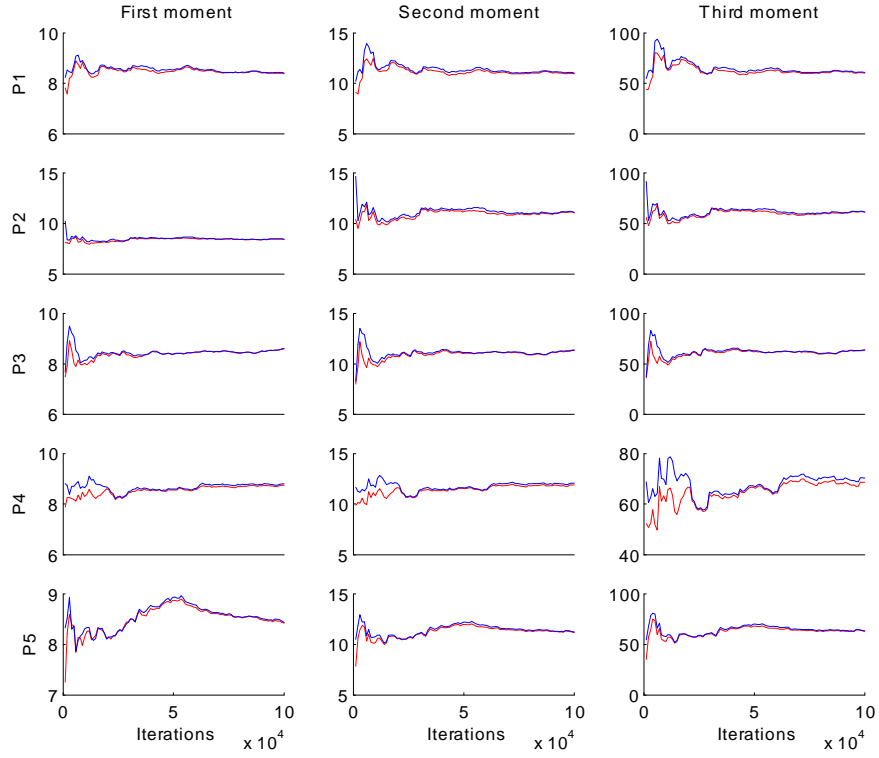
Figure 3: Multivariate Metropolis-Hastings convergence diagnosis

C   Priors and posteriors

The vertical red line denotes the posterior mode, the dashed green line the prior distribution, and the blue line the posterior distribution.
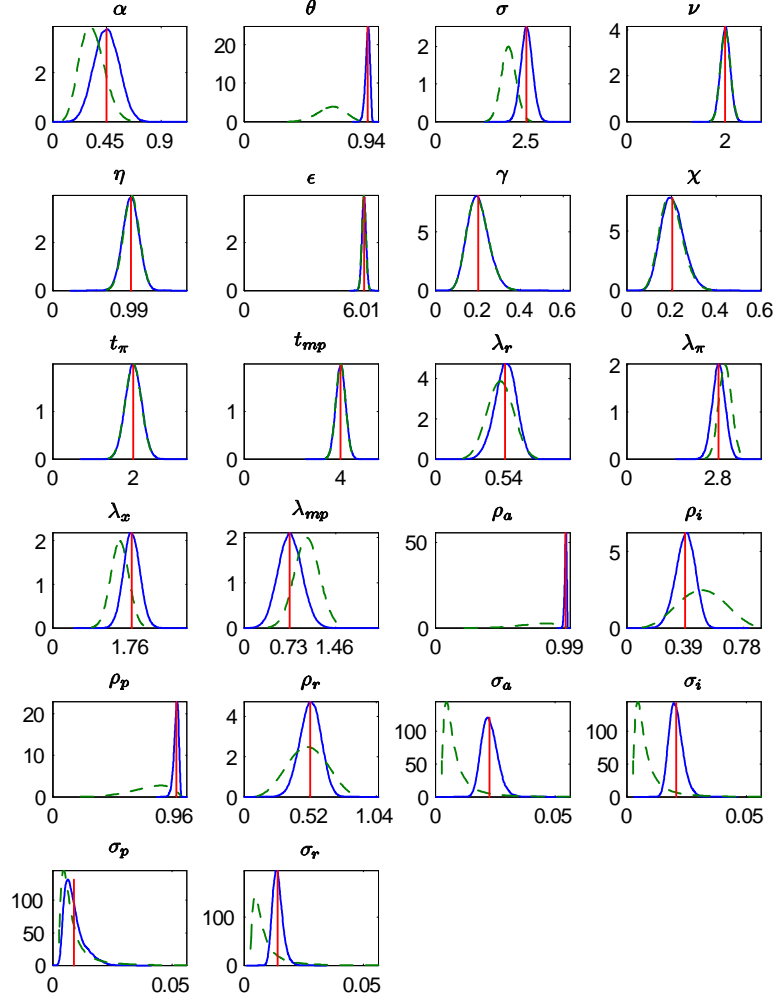
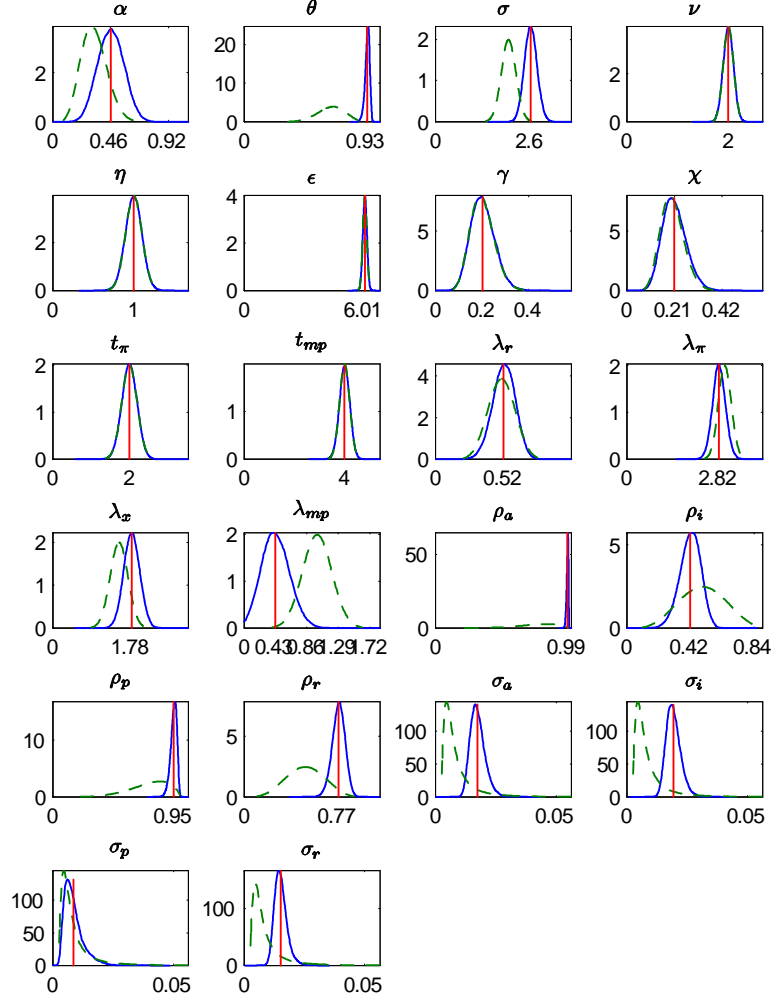Figure 4: Priors and posteriors of the estimated parameters (P1)

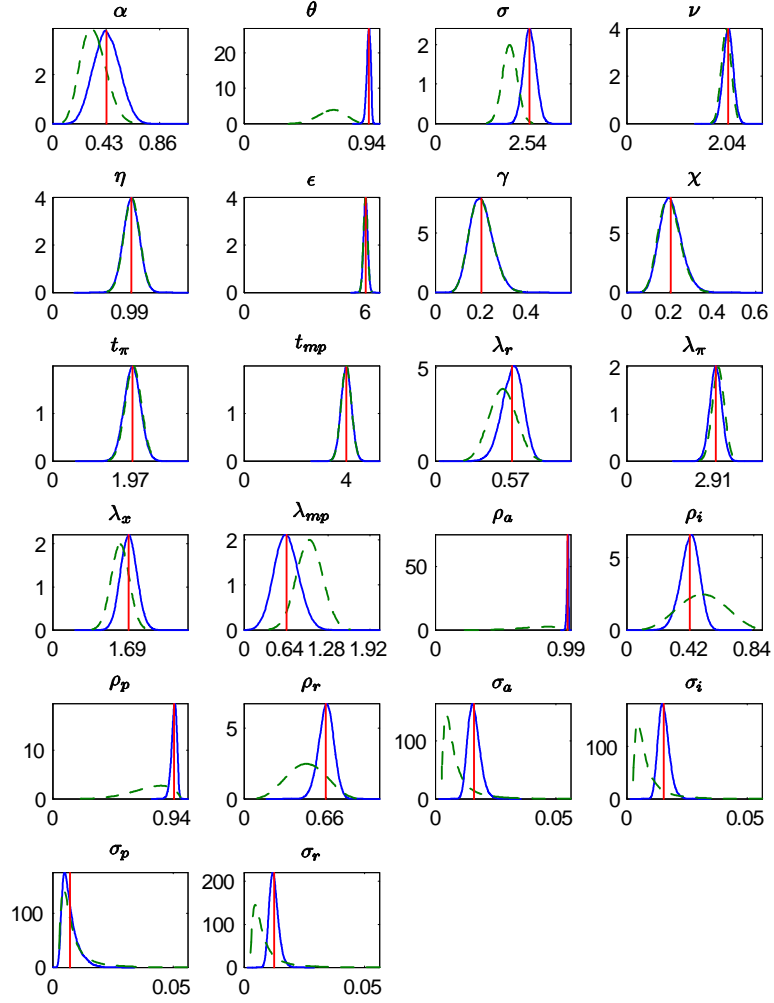Figure 5: Priors and posteriors of the estimated parameters (P2)

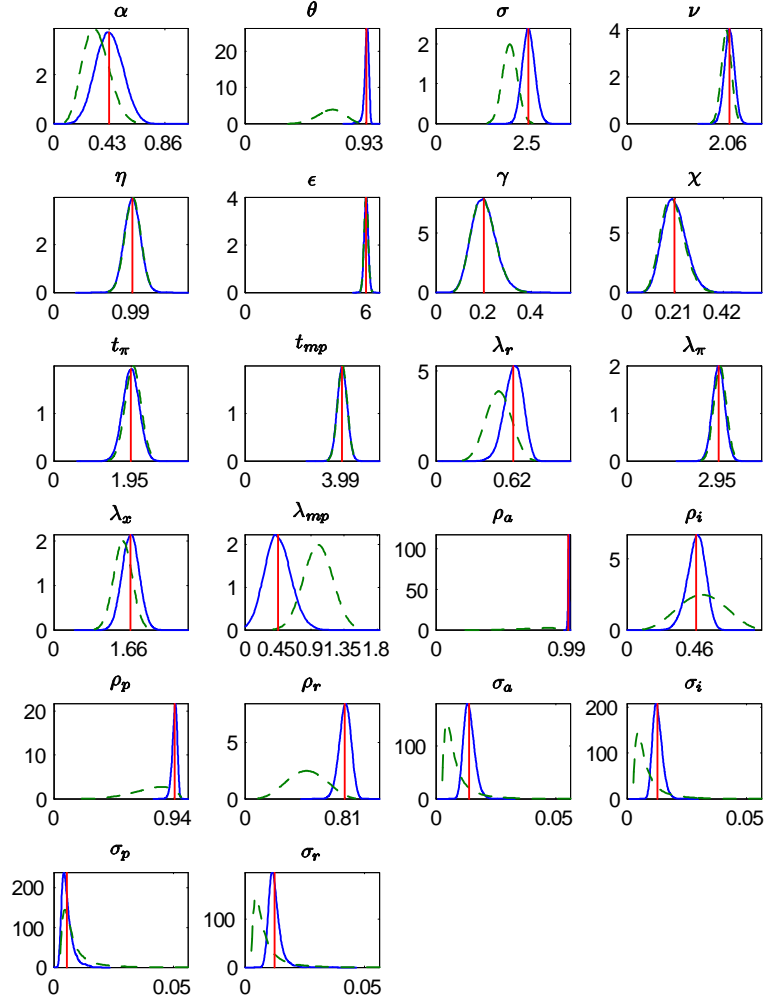Figure 6: Priors and posteriors of the estimated parameters (P3)

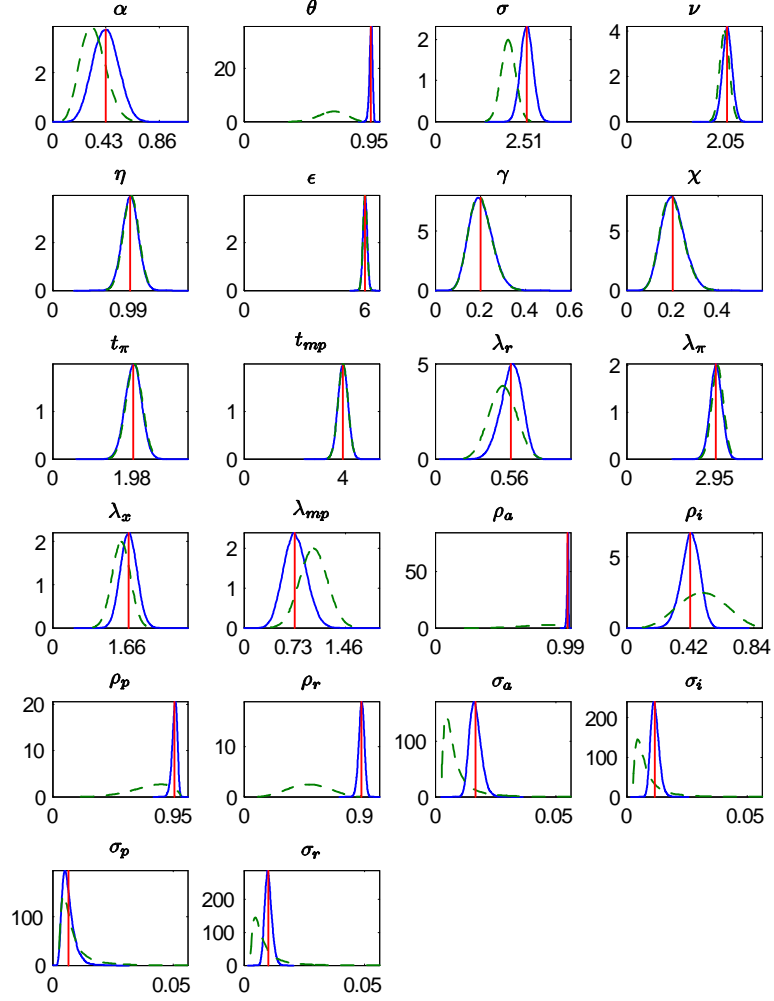Figure 7: Priors and posteriors of the estimated parameters (P4)

Figure 8: Priors and posteriors of the estimated parameters (P5)

D Impulse response functions

The black line, blue line, red line, cyan line and magenta line, represent, respectively the P5, P4, P3, P2 and P1 impulse response functions.
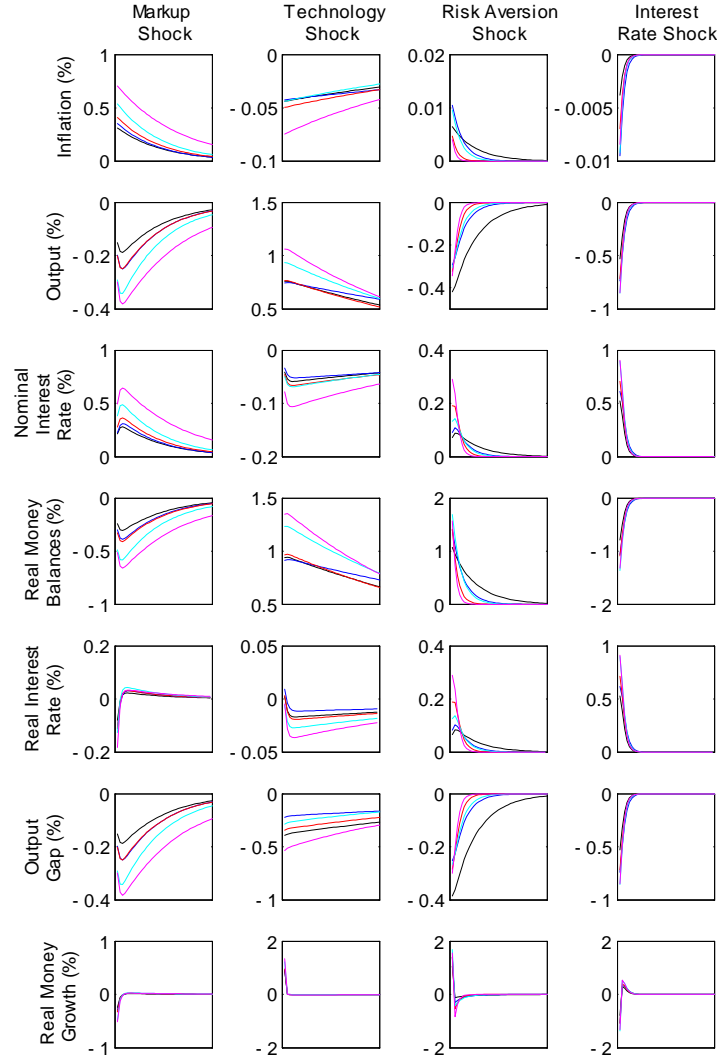

Figure 9: Impulse response functions

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