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Statistical discrimination without knowing statistics: blame social interactions? *

Emily Tanimura

Centre d'économie de la Sorbonne, Université Paris 1. †

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Abstract

We consider a model where decision makers repeatedly receive candidates and assign to them a binary decision that we can interpret as hire/not hire. The decision makers base their decision on the characteristics of the candidate but they are also sensitive to the social influence exerted by the observed past choices of their peers. We characterize the long run frequency of decisions in the model, and show in particular that for candidates belonging to a group with "unfavorable" characteristics, the dynamics increase the rejection rate compared to a scenario with independent decisions, suggesting that influence between decision makers can generate effects very similar to those that result from statistical discrimination. In our model, we then relate the long run outcomes, existence and magnitude of reinforcement to the properties of the characteristics distribution.

JEL Classification: D83 D91 J70 C60 R30

Keywords: Statistical discrimination, Social influence, Binary choice, Decision dynamics, Invariant measures, Reinforcement effects

1 Introduction

Statistical discrimination, a concept introduced by Phelps [28] and Arrow [3] (see also the review by Aigner and Cain [2]) is said to occur in hiring [1], when an employer lacking

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†<emily.tanimura@univ-paris1.fr>

[1] Hiring is the typical example in models of statistical discrimination but the latter could occur in a number of other contexts, such as the granting of loans or rental contracts

sufficiently precise information about the relevant skills of the job seeker, estimates these skills based on their statistical distribution in the group to which the agent in question belongs. In contrast with classical taste based discrimination, the decision maker need not be explicitly prejudiced, nevertheless the outcomes are discriminatory in the sense that high performing individuals in low performing groups (or groups perceived as such) would unfairly be assimilated with the typical individual of his group.

In this paper, we study a situation where decision makers similarly receive candidates repeatedly over time, and assign to them a binary decision (i.e. hire/not hire). Our assumptions about behavior and available information are almost the opposite from those that usually generate statistical discrimination : decision makers do observe individual characteristics while they ignore the statistical properties of the individual's group. However, and this is our critical assumption, they are sensitive to social influence as expressed through the decisions of their peers and/or are subject to behavioral inertia in the sense that they tend to conform to their own past decisions. Our analysis suggests that these behavioral assumptions generate outcomes that look on an aggregate level very much like statistical discrimination.

The seminal models of Phelps [28] and Arrow [3] are static. A few recent studies analyze statistical discrimination in settings that are, like ours, dynamic, although focusing on rather different issues than those explored in this paper. Blume, in [9] and [10] consider a model involving decisions of both worker and employer. Workers decide whether to make a costly investment in skills. Firms receive workers of different types and decide to hire or not based on their beliefs about the worker's skill. Kim and Loury [23] analyze a similar setting with more sophisticated forward looking behavior of agents and show the persistance of discriminatory outcomes, where the dynamics may trap members of discriminated groups in bad outcomes. Bonfiglioli and Gancia [11] consider statistical discrimination resulting from matching frictions in a dynamic setting.

We analyze the long run decision frequencies in our model, focusing particularly on the outcomes of candidates from a group whose characteristics are, in a certain sense, unfavorable. We show that for such a group, social influence between decision makers will, under certain conditions, reinforce the rejection rate in hiring, compared to a benchmark case without influence, where decision makers' hiring decisions are independent. In other words, belonging to a group with unfavorable characteristics is detrimental to the candidate, as it is the case in statistical discrimination, albeit due to rather different mechanisms. Beyond this observation, this study also elucidates how the properties of the distribution of characteristics in the candidate population affect the direction and magnitude of the reinforcement effects created by social interactions.

Our assumptions about behavior differ significantly both from those underpinning taste discrimination and those which usually leads to statistical discrimination but is more similar to the latter in the respect that it does not rely on direct discriminatory intent. The employer does need to perceive the "category" (for example male/female, white/black, younger/senior etc) of the candidate and to take into account a social norm regarding this category. His behavior thus departs from that of a perfectly neutral decision maker who is "category blind". Yet, as in the case of statistical discrimination he has no intrinsic bias against a particular category.

The existence of social conformism is widely recognized in theories of social psychology and anthropology (see the discussion in Akerlof [1]). It may repose on an intrinsic desire to adhere to the standards of one's peers which generates stable social norms, see Akerlof [1] or Bernheim [5] or emerge because individuals believe that they imitate others who are better informed (e.g. Bannerjee [4]).

The impact of social influence on decisions has been documented and studied in a variety of specific contexts, ranging, to cite just a few, from the adoption of innovations (see discussion in Young [31]), engaging in petty crime (Glaeser et al [17]) or adopting pro-environmental behaviors (Lazaric et al [24]). The mechanism seems likely also in the context of hiring. A firm might, due to pure conformity, be reluctant to employ a work force whose composition differs significantly from that of its competitors, it could imagine the social norm to contain hidden information to the effect that "if others don't hire employees of a certain category it must be for a reason". The firm could also adhere to established norms by hiring minorities, females etc at a rate comparable to that of other similar firms in order to avoid potential discrimination charges and legal actions.

Lastly, our model could also be relevant to the study of what seems, at first sight, to be a very different problem, namely that of medical procedure choice in cases where two alternative treatments are available (e.g. heart surgery vs drug treatment, cesarians vs natural childbirth etc ...). This is an area in which assumptions of social influence are perhaps less controversial than in hiring and have in fact been documented by several studies (see Phelps et al[30] or Burke et al. ([14]). Statistical analyses indicate that the treatment assigned to a patient tends to reflect not just his own characteristics but the patient demographics in his region. For example an older patient in a region with a younger population is more likely to receive heart surgery (the procedure deemed more suited to younger patients) than a similar individual in an older patient demographic group. Burke et al [15]) propose a dynamic model of this situation which focuses on explaining the emergence of geographic procedure choice patterns. The phenomenon observed in medical procedure choice bears a strong ressemblance to statistical discrimination, disregarding the terminology, since there is not, in this case, an unequivocal ranking of the two treatments.

We develop a model in which decision makers assign a binary decision to a candidate based on an influence variable and on candidate type, capturing schematically the main features of hiring in the presence of social influence, or, more generally, situations with a formally similar structure, for example the medical procedure choice problem. In the same framework we can also consider the effects of a second plausible form of influence which is not social but exerted by the decision maker's own past behavior. The arguments in favor of such an effect are similar to those underpinning social influence. Just as an employer might be sensitive to a corporate culture expressed by the types of employees hired by similar firms, he might have a tendency to maintain a personal status quo in terms of employee composition. In the case of medical procedure choice, there may, in addition, be actual, objective benefits to maintaining a procedure one has acquired familiarity with.

From a more theoretical point of view, the framework we adopt places this work in the vast literature on binary choice in the presence of social interaction. Theoretical studies such as Glaeser and Scheinkman [19], which review a number of models of this type, Glaeser et al [18] or Horst and Scheinkman [21] have highlighted general properties of a large class of models of discrete choice with social interactions in a static one shot setting. These models usually exhibit multiple equilibria, translating the possibility of different outcomes despite similar fundamentals. The strength of interactions have an important impact on the properties and the multiplicity of the equilibria. Econometric works (see for example the studies of Blume et al [7],[8])deal with problems of identification in social interaction models.

Our model is most closely related to a class of binary choice models which posit a random utility function (for a general background see Manski [27]) which depends on the decision, on social influence and on a random personal taste variable associated with each choice. The random taste variable in this formulation can also be interpreted as a random perturbation of the agent's deterministic best reply. A seminal model of this type is due to Brock and Durlauf [12] (see also by the same authors [13]) who established a link between the random utility framework and work in the area of particle spin theory in Physics. For a review of such approaches see Blume [6]. Brock and Durlauf, characterized the outcomes in their model for a large (continuum of agents) population, using the notion of correct a anticipation equilibria. The Brock and Durlauf model, or similar frameworks have been widely used and have later been extended in different ways, for example to allow for heterogenous anticipation as in Lee et al [25], or in order to accommodate more complex interaction structure than that of uniform influence e.g. Ioannides [22], Cont and Loewe [16] and Loewe et al [26].

This work can also be related to the Brock and Durlauf model, since the one-shot model that is repeated over time in our framework, coincides, for a specific choice of the utility function, with the Brock and Durlauf model. In this particular case, we can also identify the distribution of the difference between the taste variables associated with each action in Brock and Durlauf with the characteristics distribution in our model.

However, in general, our model differs from the Brock and Durlauf model and its extensions in that our decision makers make decisions about an exterior population and not about themselves. This difference in context also prompts new questions. In the random utility framework, the random term, interpreted as a perturbation of best reply, is essentially a noise term and generally not the object of much attention, an exception being Gordon et al [20] who analyzed the Brock and Durlauf model for a more general class of distributions of the random term. In our case, however, the random term represents the characteristics of the population. It is thus a crucial aspect of the model and it is important to understand what is the impact of its properties on the long run outcomes.

1.1 Organization of the material

Rather than making from the start restrictive assumptions motivated mainly by the need to simplify the mathematical analysis, we initially adopt a quite general and flexible framework in which we may consider different types of preferences of the decision maker, candidate characteristics distributions and interaction structures. It is difficult to obtain results about the model in its full generality. We explore two special cases, one with general interaction structures but restrictive assumptions on the utility functions and the distribution of candidate characteristics and another in which we fix a simple interaction structure while allowing utility and characteristics specifications to vary.

Following the introduction in section 1, section 2 presents the general framework and assumptions. In section 3, we characterize average long run decision frequencies for an arbitrary interaction structure restrictions on the utility functions and the distribution of candidate characteristics . In section 4, we characterize decision frequencies for arbitrary characteristics distributions but only for two very particular interaction/influence structures: uniform interaction between decision makers and pure personal reinforcement. We show that long run decision frequencies can be related to the fixed points of the choice probability.

Section 5 does not present any new results. It provides a discussion of the results in sections 3 and 4, focusing on how to distinguish between the effects of social interactions or personal inertia when we observe only aggregate data.

Section 6 uses the results in 4 to analyze how the magnitude of the asymptotic reinforcement of a decision is related to various properties of the characteristics distribution.

Section 7 conclues. An appendix at the end of the paper presents longer proofs that are left out from the main text.

2 Framework and assumptions

2.1 Utility and decisions

N decision makers with identical preferences face a binary decision between two choices labeled 0 and 1. If we interpret the decision as whether or not to hire the candidate, we can, from now on, think of decision 1 as rejection. The utility function of decision maker i is noted U_i and depends on three arguments: the decision $a_i \in \{0,1\}$, the candidate characteristic θ and an influence variable $h_i \in [0,1]$ that can encompass social influence and personal inertia and which will be defined more precisely later on. The candidate characteristic $\theta \in \mathbf{R}$ comprises all relevant personal characteristics of the candidate, such as experience, motivation etc . We adopt the convention that a higher value of θ makes the candidate more suited for decision 1 (rejection). For each candidate, θ results from an independent draw of the random variable Θ whose distribution describes the characteristics of the candidate population. The decision maker takes action 1 if $U_i(1,h,\theta) > U_i(0,h,\theta)$ (indifference is settled by the draw of an unbiased coin). We make the following assumptions about the utility function:

(1) $U_i(1,h_i,\theta) - U_i(0,h_i,\theta)$ is strictly increasing in h_i and in θ. This translates the convention that a higher θ makes the candidate more suitable for decision 1 and the fact that a higher value of the influence variable makes decision 1 more attractive compared to decision 0.

(2) Presence of dominant candidate types: there is a θ_h such that for $\theta \geq \theta_h$, $U_i(1,h,\theta) > U_i(0,h,\theta)$ for all $h \in [0,1]$ and there is a θ_l such that for $\theta \leq \theta_l$ $U(0,h,\theta) > U(0,h,\theta)$ for all $h \in [0,1]$.

(3) Let $\theta_l < \theta_n < \theta_h$ be the "neutral" candidate, the type that makes the decision maker indifferent between choices when the environment is unbiased. It is defined by: $U_i(1,\frac{1}{2},\theta_n) = U_i(0,\frac{1}{2},\theta_n)$. The decision maker has no personal preference over decisions in the sense that for all $h_i \in [0,1]$ and all $\alpha > 0$, $U_i(1,h_i,\theta_n+\alpha) = U_i(0,1-h_i,\theta_n-\alpha)$.

We adopt a dynamic model in discrete time. At $t \geq 1$, each decision maker i receives a candidate and makes a decision $a_i^t \in \{0,1\}$ so as to maximize his utility function $U_i = U_i(a_i^t,\theta,h_i^t)$.

2.2 Social or Personal Influence

A key assumption in this paper is that in addition to the candidate's characteristics, the decision maker takes into account a second factor that can be social influence,

influence of his own pas choices or a combination of both. This is captured in the variable h_i^t. Most generally, h_i^t will be a weighted average of a neighbor variable v_i^t and a personal experience variable p_i^t, so that $h_i^t = \gamma v_i^t + (1-\gamma)p_i^t$, $\gamma = 1$(or 0) corresponds to pure social influence and pure personal reinforcement respectively. The neighbor variable h_i^t reflects the proportion of the agent's neighbors who have chosen the action 1 in the past. If we denote by a_i^t, the action of agent i at time t and by $V(i)$, i's neighbors in the network Γ, then the neighbor state variable is updated by adding the average of the most recent actions of the agent's neighbors. It is thus defined recursively by $v_i^{t+1} = \lambda_2 v_i^t + (1-\lambda_2)\frac{1}{CardV(i)}\sum_{j\in V(i)} a_j^{t+1}$ (assuming identical impact of all neighbors' choices), where λ_2 is the peer discount factor. Similarly, the agents' own past experiences are summarized by $p_i^{t+1} = \lambda_1 p_i^t + (1-\lambda_1)a_i^{t+1}$, where λ_1 is the discount factor for personal experience. Updating rates of own observations and of those of one's peers may differ.

2.3 The distribution of candidate characteristics

All characteristics of the candidate that are relevant for the decision (0 or 1) are summarized by a scalar $\theta \in R$ which is the realization of random variable Θ.The variable Θ captures the distribution of characteristics in the group the agent belongs to. To study the effects of social interactions on the outcomes for a group with an initial bias towards one of the decisions, we need to define what is meant by an initial bias. Without loss of generality, we will speak about bias in favor of decision 1. Bias in favor of decision 0 would be defined in an analogous manner. If we think of hiring, 1-bias in a population expresses a lack of job relevant education, experience etc in the group. With binary characteristics, there would quite clearly be (1) dominance if more than half of the agents were of a higher type than the neutral type θ_n . With continuous candidate types it is less straightforward to define what is meant by dominance. Two possible definitions are:

(1) (weak bias) $E[\Theta] > \theta_n$

(2) (strong bias)for all $\alpha > 0$, $P([-\infty, \theta_n - \alpha]) < P([\theta_n + \alpha, \infty])$

2.4 Main assumptions in terms of choice probability

By the previous, the decision maker chooses 1 if and only if

$$U_i(1, h_i^t, \theta) \geq U_i(0, h_i^t, \theta) \tag{1}$$

Ex ante, the probability of choosing 1 is given by

$$P(U_i(1, h_i, \theta) - U_i(0, h_i, \theta) \geq 0) =: P(1|h_i). \tag{2}$$

In what follows, we will often work directly with the choice probabilities and when needed, go back, to translate the results in terms of utility and characteristics. The assumptions about preferences and characteristics that we stated in the previous sections translate into the first 3 properties below of the choice probability. In addition, we assume property 4, which holds if the utility function is continuous in each of its arguments and if the cdf of Θ has at most a finite number of discontinuities.

(1) (strong bias) $P(1|h) > 1 - P(1|1-h)$ for all $h \in [0, 1]$.

(2) (dominant types) $P(1|1) < 1$, $P(0|0) < 1$.

(3) $P(1|h)$ is increasing in h

(4) $P(1|h)$ is continuous or admits a finite number of discontinuities.

2.5 Dynamics and long run quantities of interest

Every period $t = 1, 2..$, each decision maker receives a candidate drawn from the candidate distribution and assigns him a decision. The number of 1 choices at time t is thus $\sum_{i=1}^{i=N} a_i^t \in \{0, 1, ..., N\}$ and the expected fraction of 1 decisions is $E[\frac{1}{N}\sum_{i=1}^{i=N} a_i^t]$. We will focus on cases where $\sum_{i=1}^{i=N} a_i^t$ converges asymptotically to a distribution μ on $\{0, 1, ..., N\}$ which we can characterize, or on cases where we can at least characterize the limit expectation $\lim_{t\to\infty} E[\frac{1}{N}\sum_{i=1}^{i=N} a_i^t]$. These quantities then give us the long run distribution of 1 choices in the presence of influence, and/or the expected frequency of 1 choices. To evaluate the reinforcement, we must compare this to the decisions that would be taken in absence of social influence. It is not possible to remove the influence variable h from the model. To eliminate influence, instead, we fix the influence variable h permanently at the neutral value $h = \frac{1}{2}$. The decision makers' probabilities of choosing 1 are then independent and identically equal to $P(1|\frac{1}{2}) = P(\Theta_1 \geq \theta_n)$ over time. Let ν be the distribution over the states $\{0, .., N\}$ (a state can be identified with the number of 1 choices taken) in absence of interactions. ν is then a binomial distribution with parameter $P(1|\frac{1}{2})$, which is also the expected value of the fraction of 1 decisions in absence of social influence. The change in frequency of 1 choices for a population with characteristics given by Θ due to social influence can thus be measured by

$$lim_{t\to\infty} E_\Theta[1/N \sum_{i=1}^{i=N} a_i^t)] - P(\Theta_{\geq}\theta_n) = lim_{t\to\infty} E_\Theta[1/N \sum_{i=1}^{i=N} a_i^t)] - P(1|\frac{1}{2}) \tag{3}$$

When $\sum_{i=1}^{i=N} a_i^t$ converges to a distribution μ, the latter can be compared to the one in absence of interactions, ν. Also, in this case, $lim_{t\to\infty} E_{\Theta_1}(\frac{1}{N}\sum_{i=1}^{i=N} a_i) = \frac{1}{N}\sum_{i=1}^{i=N} \mu(i)$.

We may also want to compare how social influence impacts two groups with different characteristics. Let Θ_1 and Θ_2 be the characteristics distributions in the two groups. Assuming convergence to measures μ_1 and μ_2 respectively, the difference in long run outcomes is then $lim_{t\to\infty}[E_{\mu_1} 1/N \sum_{i=1}^{i=N} a_i) - E_{\mu_2} 1/N \sum_{i=1}^{i=N} a_i)]$ but even in absence of interactions, the fractions of 1 decisions were not necessarily the same. They would be $P(\Theta_1 \geq \theta_n)$ and $P(\Theta_2 \geq \theta_n)$ respectively. Thus the reinforcement of the difference in outcomes for the two groups is measured by

$$\lim_{t\to\infty}[E_{\mu_1} 1/N \sum_{i=1}^{i=N} a_i) - E_{\mu_2} 1/N \sum_{i=1}^{i=N} a_i] - [P(\Theta_1 \geq \theta_n) - P(\Theta_2 \geq \theta_n)] = \quad (4)$$

$$(lim_{t\to\infty} E_{\Theta_1}(\frac{1}{N}\sum_{i=1}^{i=N} a_i^t) - P(\Theta_1 \geq \theta_n)) - (lim_{t\to\infty} E_{\Theta_2}(\frac{1}{N}\sum_{i=1}^{i=N} a_i^t) - P(\Theta_2 \geq \theta_n)) \quad (5)$$

Note that this quantity can be deduced from 3 . We will let the expression 3 be our main measure of reinforcement.

3 Special case 1: general interaction structure and influence structure, uniform characteristics distribution

In this section, we analyze a case where we place very few restrictions on interaction and influence structures but impose very specific assumptions about preferences and the characteristics distribution. We consider a utility function of the form

$$U_i(a_i, h_i, \theta) = \beta(h_i - \frac{1}{2})(a_i - \frac{1}{2}) + (\theta - \frac{1}{2})(a_i - \frac{1}{2}).. \quad (6)$$

This utility specification is similar to one of those in [19] but adapted to our actions (0 and 1 instead of -1, 1, neutral value $\frac{1}{2}$ instead of 0). The parameter β regulates the strength of social influence. We note that the neutral type is then $\theta_n = \frac{1}{2}$. Moreover, we suppose that the characteristics follow a uniform distribution on an interval of length 1 centered in P_c with $0 < P_c < 1$. With these specifications, the choice probability is linear in the influence variable. Ex ante, the probability of choosing 1 for a given value h_i of the state variable is:

$$P(a_i = 1|h_i) = P(U(1) \geq U(0)|h_i) = P(\theta \geq \frac{1}{2}(\beta + 1) - \beta h_i) = \quad (7)$$

$$P_c + \frac{1}{2} - \frac{1}{2}(\beta + 1) + \beta h_i = P_c + \beta(h_i - \frac{1}{2}) \quad (8)$$

P_c is the proportion of 1 choices in a neutral environment. Strong 1-dominance is ensured if $P_c > \frac{1}{2}$.

Our first result characterizes the long run behavior of the expectation of the aggregate quantity $V = \frac{1}{N}\sum_{i=1}^{i=N} v_i$, the average of the neighbor variables and of $P = \frac{1}{N}\sum_{i=1}^{i=N} p_i$, the average of the agents' own past choices. From these, is easily deduced the expected long run frequency of 1 decisions.

Theorem 1 *Assuming that*

(1) utility is given by (6)

(2) the characteristic Θ *follows a uniform distribution on an interval of length 1 centered in* P_c *with* $0 < P_c < 1$*.*

(3) The agents are connected in a network Γ *in which all agents have the same degree.*

(4) social/personal influence is limited: $\beta < \frac{1-P_c}{2}$

(5) 1 *dominates in the population in the sense that* $P_c > \frac{1}{2}$

Then the choice probability has a unique fixed point in $[0,1]$, $x_f = \frac{P_c - \frac{\beta}{2}}{1-\beta} > P_c$ *and for all choices of* γ, λ_2 *and* λ_1 *in* $(0,1)$ *we have*

$$\lim_{t \longrightarrow \infty} E_{V^0}[V^t] = \lim_{t \longrightarrow \infty} E_{P^0}[P^t] = x_f$$

for any initial conditions V^0 *and* P^0*.*

The proof of Theorem 1 is given in the appendix. The result shows that in the long run, the expected values of the private and public state variables are the same and equal the fixed point whatever are the values of the updating parameters and the weight given to private and public experience respectively. This is of course the case only for these specific choices of utilities and characteristics distribution.

Corollary 2 *Under the assumptions of Theorem 1 the long run average of 1-decisions* $lim_{t\to\infty} E[1/N \sum_{i=1}^{i=N} a_i^t)] = x_f = \frac{P_c - \frac{\beta}{2}}{1-\beta} > P_c$*. The long run reinforcement is* $x_f - P(\Theta \geq \theta_n) = x_f - P_c$ *which is strictly positive when* $P_c > \frac{1}{2}$*.*

The corollary shows that when the population characteristics verify 1-bias (i.e. unfavorable characteristics), interaction between decision makers reinforces the frequency of 1 decisions (candidate rejected) compared to the case with independent decisions.

Proof of Corollary 2 : If, for example, the weight on personal experience is not 0 (if it is the weight on neighbors past actions that is non zero the argument is identical), we have $p_i^t = \lambda p_i^{t-1} + (1-\lambda)a_i^t$. Thus by Theorem 2

$$lim_{t\to\infty} E[1/N \sum_{i=1}^{i=N} a_i^t)] = \frac{1}{N(1-\lambda)} lim_{t\to\infty} E[\sum_{i=1}^{i=N} p_i^t) - \lambda \sum_{i=1}^{i=N} p_i^{t-1})] = \tag{9}$$

$$\frac{1}{(1-\lambda)} lim_{t\to\infty} E[P^t - \lambda P^{t-1}] = x_f \tag{10}$$

Although the results above are obtained under very specific assumptions about functional forms, they already indicate that social interactions between decision makers can give rise to effects resembling those of statistical discrimination. If we take decision 1 to mean that the candidate is rejected at hiring, we see that with influence between decision makers, individuals from a group with unfavorable characteristics will face a lower rate of hiring than in absence of the interactions. We also see that these reinforcement effects can be explained by two different behavioral assumptions: social interactions, personal inertia, or a combination of both.

4 Special case two and three: Pure social influence with short memory and Pure personal inertia

In the previous section, we placed few restrictions on interaction and influence structure (any network with identical degrees, any mix of social and personal reinforcement and updating speed) but imposed specific functional forms for preferences and characteristics distributions. We now do the opposite: limiting our analysis to two particular influence structures but allowing preferences and characteristics distributions to be arbitrary among those satisfying our general assumptions. This will allow us to study how the distribution of candidate characteristics influences the long run outcomes and in particular the magnitude of discriminatory reinforcement. We restrict attention to the following two situations :

- Uniform interaction (ie on a complete graph) and memory of only the most recent period. This corresponds to setting updating parameter $\lambda_2 = 1$. We set $\gamma = 1$, meaning that no weight is given to personal experience. For reasons of analytical convenience, we assume here asynchronous updating. At discrete times, an agent $i \in \{1, 2, ..., N\}$ is drawn uniformly at random and receives a candidate. He makes a decision based on the candidate type and on the average choice in the last period, $\frac{1}{N-1} \sum_{i=1}^{i=N-1} a_i^{t-1}$.

- Pure personal reinforcement. The decision maker cares only about his own previous choices ($\gamma = 0$). Again, for reasons of analytical convenience and to obtain an identification with the first case, we consider a slightly different updating mechanism than in the previous framework : the agent remembers his own N most recent decisions, which are summarized in a memory vector $p \in \{0,1\}^N$. At each time t the agent's choice is affected by the average value of his memory vector. Thus at time t the agent chooses 1 with probability $P(1|p^{t-1})$ where $p^{t-1} = \frac{1}{N}\sum_{s=t-1-N}^{s=t-1} a^s$ is the average value of his memory vector. He updates this vector by adding his choice at time t and by dropping the most ancient choice still in memory, v_{t-N}.

In the first case, $S_N^t =: \sum_{i=1}^{i=N} a_i^t$, the sum of all agents decisions is an ergodic, homogeneous Markov chain on the state space $\{0,1\}^N$, and so is, in the second case, $R_N^t =: \sum_{l=t-N}^{l=t} a_i^l$, where N represents in the first case the number neighbors of the decision maker and in the second case the number of periods that the agent remembers. We note that in the general case, the sum of decisions is not Markovian due to dependance on past actions in the personal and neighbor variable.

Proposition 3 *The variables (S_N^t) and (R_N^t) have the same invariant measure μ with*

$$\mu(k) = \frac{\begin{pmatrix} N \\ k \end{pmatrix} \prod_{i=0}^{i=k-1} \frac{P(1|\frac{i}{N})}{P(0|\frac{i+1}{N})}}{\sum_{l=1}^{l=N} \begin{pmatrix} N \\ l \end{pmatrix} \prod_{i=0}^{i=l-1} \frac{P(1|\frac{i}{N})}{P(0|\frac{i+1}{N})}}.$$

Proof : We give the proof for R_N. For S_N, the proof can be found in the appendix. We adopt the generic notation v_k for any $v \in \{0,1\}^N$ such that $\frac{1}{N}\sum_{i=1}^{i=N} v_i = k$. As an ergodic, homogenous Markov chain, (R_N^t) admits a unique invariant measure. We seek an invariant distribution that puts the same probability on all elements with the same mean and write the detailed balance conditions for an element v_k. There are two possible cases: if the most recent element in the memory vector v_k is 1, then the most recent decision was 1. There are two possible precursors of this vector, one where the dropped element was 1 and one where it was 0. If the dropped element was 1, the unique precursor is an element v_k, if the dropped element was 0 the unique precursor is an element v_{k-1}, since the updating replaced a 0 by a 1 :

$$\mu(v_k) = \mu(v_k)P(1|\frac{k}{N}) + \mu(v_{k-1})P(1|\frac{k-1}{N}) \tag{11}$$

In the second case, the most recent entry of the vector v_k is 0. In this case, there is a unique precursor v_k, whose most ancient entry is 0 and a unique precursor v_{k+1} whose most ancient entry is 1. For the element v_k such that the last entry is 0 :

$$\mu(v_k) = \mu(v_k)P(0|\frac{k}{N}) + \mu(v_{k+1})P(0|\frac{k+1}{N}) \tag{12}$$

In both cases, the equations simplify to

$$\mu(v_k) = \frac{\mu(v_{k-1})P(1|\frac{k-1}{N})}{P(0|\frac{k}{N})}. \tag{13}$$

Recursively we find that

$$\mu(v_k) = \prod_{i=0}^{i=k-1} \frac{P(1|\frac{i}{N})}{P(0|\frac{i+1}{N})}\mu(v_0) \tag{14}$$

Since μ is a probability, and since there are C_k^N different elements whose sum is k,we must have $\mu(0) = \frac{1}{1+\sum_{k=1}^{k=N} C_k^N \prod_{i=0}^{i=k-1} \frac{P(1|\frac{i}{N})}{P(0|\frac{i+1}{N})}}$.

4.1 The nature of the reinforcement effects

Having derived the expression for the invariant distribution μ of decisions in the presence of social influence/inertia, we can analyse the effects of the latter by comparing with the decisions that would have been taken in absence of these factors. We recall from section 2.5 that without influence, the distribution on the states $\{0, 1, ..., N\}$ (representing the number of 1 decisions) is a Binomial law $Bin(N, p)$ with parameter $p = P(\Theta_1 \geq \theta_n) = P(1|\frac{1}{2})$.

4.1.1 Asymptotic analysis; large number agents/ long memory

The lemma below will be helpful in characterizing the behavior of μ for large N.

Lemma 4 *Consider the function*

$$g(x) = \frac{P(1|x)(1-x)}{(1-P(1|x))x}. \tag{15}$$

Let $c > 0$, then for any $\frac{k}{N} \in [c, 1]$, we have

$$g(\frac{k+1}{N}) - R(N) < \frac{\mu(k+1)}{\mu(k)} < g(\frac{k}{N}) + \tilde{R}(N) \tag{16}$$

where $\lim_{N\to\infty} R(N) = \lim_{N\to\infty} \tilde{R}(N) = 0$.

When N is large, μ is increasing when $g > 1$ and decreasing when $g < 1$.

The next proposition describes the invariant measure when there is a strong bias towards type 1 and under an assumption on the fixed points of the choice probability.

Theorem 5 *If we have strong bias towards type* 1 *and if x is the only fixed point of $P(1|x)$ in $[\frac{1}{2}, 1]$, and if we define $A = N(x - \epsilon, x + \epsilon)$ then for every $\epsilon > 0$ and every M, there exists $N(\epsilon, M)$ such that $\frac{\mu_N(A)}{\mu_N(A^c)} > M$.*

We can note that when the characteristics distribution has a continuous cumulative density function, the strong bias condition and the assumption of existence of dominant types already ensures the existence of a fixed point in $[\frac{1}{2}, 1]$, and the assumptions in Theorem 5 is just needed to ensure uniqueness. Theorem 5 is proved in the appendix. It provides a clear characterization of the magnitude of the reinforcement effects generated by social influence when the population size is large, since the average under μ_N is given by the fixed point which is larger than the average in the absence of interactions, since $x_f > P(1|\frac{1}{2})$. When N is sufficiently large, and the characteristics distribution verifies the condition of having a unique fixed point in $[\frac{1}{2}, 1]$, then social interactions do indeed increase the frequency of the treatment towards which there is an initial bias.

4.1.2 Pre-asymptotic analysis: small number of agents/short memory

The previous result is asymptotic. Away from the large N limit, the distribution μ puts positive weight on all states and there is a wider range of properties to explore.

The first proposition concerns the case where there is no bias in the population initially.

Proposition 6 *Assume that there is no bias in the candidate characteristics. Then social interactions/inertia do not modify the average treatment: $E_\mu(X) = E_\nu(X)$ but increase variability : there is second order stochastic dominance of ν over μ.*

In the presence of interactions (or inertia), the probability of extreme outcomes, that is outcomes where many decision makers take similar decisions simultaneously is increased. This result is quite intuitive and is a dynamic equivalent to the observation that social influence generates multiple equilibria in one shot models.

Proof: In this case, μ and ν are both symetricly distributed around $N/2$. Consequently, $\sum_{k=0}^{k=N/2} \mu(k) = \sum_{k=0}^{k=N/2} \nu(k) = \frac{1}{2}$. Suppose that $\mu(0) < \nu(0)$. Then for all $0 < k \leq N/2$ we would have $\mu(k) = \mu(0) c_k^N \prod_{j=0}^{j=k-1} \frac{P(1|j)}{P(0|j+1)} < \nu(k) = \nu(0) c_k^N (\frac{p}{1-p})^k$, since $P(1|k) < p = P(1|\frac{N}{2})$ for $k < N/2$. This contradicts $\sum_{k=0}^{k=N/2} \mu(k) = \sum_{k=0}^{k=N/2} \nu(k)$. Therefore, $\mu(0) > \nu(0)$. For the same reason, we cannot have for all $0 < k \leq N/2$ $\mu(k) > \nu(k)$. Therefore, there exists a smallest $m < N/2$ such that $\mu(m) \leq \nu(m)$.

For $m < k < N/2$, we have $\mu(m) < \nu(m)$. By symmetry, we have $\mu(k) - \nu(k) = \mu(N-k) - \nu(N-k) > 0$ on $k \in [0, ..., m-1]$ and $\mu(k) - \nu(k) = \mu(N-k) - \nu(N-k) < 0$ on $k \in \{m, ..., N-m\}$. It is apparent from the shape of the two distributions that μ is a mean preserving spread of ν.

Next, we turn to the main case with which we are concerned in this paper, the one where the population has characteristics biased towards 1. The following proposition shows, somewhat surprisingly that for a fixed N, even strong 1-bias does not necessarily ensure that social interactions have for effect to increase the frequency of decision 1. We can exhibit distributions of characteristics for which the opposite occurs.

Proposition 7

- *For every fixed $N \geq 2$, we can find a characteristics distribution P verifying strong bias under which the distribution μ_P is stochastically first order dominated by ν.*
- *For every fixed $N \geq 2$, there exists a characteristics distribution Q verifying strong bias under which the distribution μ_Q stochastically first order dominates ν. A sufficient condition for this to occur is that $P(1|1) >> P(1|1/2)$.*

We will work with the conditional probability distribution $P(1|x)$ and prove the first claim by exhibiting a choice probability for which $\nu DS1\mu$: Let $P(1|x) = p > 0,5$ for $x \geq 1/2$ and let $P(1|x)$ be arbitrary for $x < 1/2$ but verifying the strong bias condition, ie $P(1|x) > 1-p$. The invariant distribution verifies for $k \leq N/2$: $\mu(k) = c_k^N \prod_{i=0}^{k-1} \frac{P(1|\frac{i}{N})}{P(0|\frac{i+1}{N})}\mu(0)$, and for $k > N/2$, $\mu(k) = \frac{c_k^N}{c_{N/2}^N} \prod_{i=0}^{k-N/2} \frac{P(1|\frac{\frac{N}{2}+i}{N})}{P(0|\frac{\frac{N}{2}+i+1}{N})}\mu(N/2) = \mu(N/2)(\frac{p}{1-p})^{k-N/2}$

Now, ν is a $Bin(N,p)$ law, so $\nu(k) = c_k^N \frac{p}{1-p}\nu(0)$. For $k \geq N/2$, we can write $\nu(k) = \frac{c_k^N}{c_{N/2}^N}(\frac{p}{1-p})^{\frac{N}{2}-k}\nu(N/2)$.

We show first that it is not possible that $\mu(0) \leq \nu(0)$ because this would imply that for all $m \leq N/2$, $\mu(m) < \nu(m)$. But if $\mu(N/2) < \nu(N/2)$, then we would also have $\mu(k) < \nu(k)$ for all $k > N/2$, which is impossible since $\sum \mu(k) = \sum \nu(k)$.

Thus, necessarily, $\mu(0) > \nu(0)$. Then there must exist a smallest $k \in \{1, ..., N/2\}$ such that $\mu(m) < \nu(m)$ for all $m > k$. Indeed if such a k did not exist then $\mu(N/2) > \nu(N/2)$ would imply $\mu(k) > \nu(k)$ for all k which is impossible. This concludes that for the given probability, we have $\mu(k) > \nu(k)$ for all $k < m$ and $\mu(k) < \nu(k)$ for $k > m$ and indeed $\nu DS1\mu$ as we claimed. We note that under the same assumptions, the fixed point which measures increase asymptotically is exactly equal to $P(1|N/2)$ so that no increase occurs.

We prove the second point: we recall that $\mu(0) = \frac{1}{1+\sum_{k=1}^{k=N} C_k^N \prod_{i=0}^{i=N-1} \frac{P(1|\frac{i}{N})}{P(0|\frac{i+1}{N})}}$. Now,

$\prod_{i=0}^{i=N-1} \frac{P(1|\frac{i}{N})}{P(0|\frac{i+1}{N})} = \prod_{i=1}^{i=N-1} \frac{P(1|\frac{i}{N})}{P(0|1-\frac{i}{N})} \frac{P(1|0)}{P(0|1)} \geq \frac{P(1|0)}{P(0|N)}$. For a choice probability such that $P(1|1)$ is sufficiently large compared to $p = P(1|\frac{1}{2})$, we can ensure that $\mu(0) < (1-p)^N$

By arguments similar to those in the previous proof, this implies that for all $k \leq N/2$, $\mu(k) < \nu(k)$. Moreover, there must exist a smallest $k \in \{N/2+1, ..., N\}$ such that for $m > k$, $\mu(m) > \mu(k)$. This concludes the proof that there exists a distribution such that $\mu DS1\nu$.

Proposition 7 shows that, away from the asymptotic limit, social influence or inertia, may in some cases, reinforce rejection rates in a population whose characteristics verify strong initial bias but in others reduce it. Since stochastic first order dominance is not a complete order, it is also possible that the effect is undetermined for some characteristics distributions, so that μ does not first order dominate ν nor conversely. We have related the effect that obtains to properties of the probability $P(1|x)$. It is instructive to see how the latter translate into properties of the characteristics distribution. The second part of Proposition 7 says that reinforcement occurs if $P(1|1) >> P(1|1/2)$. This will be the case if $P(1|1/2)$ is close to $\frac{1}{2}$ and then increases sharply with x. This holds if there is only a minor bias towards 1 in the population but a very small proportion of dominant 0 types. The counter examples that give the first part of Proposition 7 are such that $P(1|1/2)$ is well above 1/2 but barely increases from there and $P(1|x)$ is increasing on $[0, \frac{1}{2}]$: there are many agents with moderately 1 leaning characteristics but also many 0-dominant types.

5 Distinguishing between Personal inertia or social interactions

This section does not present any new results but analyzes the implications of the results of sections 3 and 4. The main objective is to clarify the differences and similarities in the outcomes that are generated under assumptions of social influence and inertia respectively. In section 4, we studied the properties of the invariant probability distribution μ, where $\mu(k)$ represents in the case of social interactions, the probability that exactly k agents choose treatment 1, this probability being the same at any given time t. In the case of personal inertia, $\mu(k)$ is the probability that an agent has chosen treatment 1 k times in the past N periods, this probability being independent across all agents.

With access to detailed individual data, it should always be possible to identify the relevant explanation. Personal inertia produces correlations over time in each individual's behavior, whereas social interactions give rise to correlations between agents connected in a network at a given time, provided we know the network. Using only aggregate data, the relevant explanation is more difficult identify. Both behavioral assumptions generate reinforcement effects of similar magnitude. Let us therefore summarize what the "aggregate" data would look like under the different assumptions.

- Social interactions increase the variance of the average choice and thus the probability of extreme scenarios where many of the decision makers take the same decision at a given moment. This can occur even with a candidate population whose characteristics distribution exhibits no initial bias. in this case, however, the probability of both types of extreme scenarios increase symmetrically. Social interactions could also be at the origin of occasional observations of large differences in treatment between two groups that do not differ in their statistical characteristics, but in this case, these differences would not show a consistent tendency over time.

- If the characteristics distribution has no bias, personal inertia, contrary to interactions, cannot account for the appearance of extreme scenarios, even without a consistent tendency. Due to the independence between decision makers, the probability that k agents choose treatment 1 is a binomial law $B(n, p)$ where p is the individual probability of choosing 1. This probability is $E_\mu(X) = P(1|\frac{1}{2})$. Therefore the probability of a scenario where k agents simultaneously choose 1 is the same as in absence of inertia.

- If the characteristics distribution is biased : the expected average long run frequency of 1-choices is modified to $E_\mu(X)$, thus with the same magnitude, both with social interactions and with inertia. Thus both behavioral assumptions produce scenarios with reinforced differences between groups differing at the outset. However, the variance over time of average choices should be greater with social interactions. In the inertia case, the individual decisions are independent and so, when the number of decision makers is large, the average choice should be close to its expectation. However, when memory length is large and the number of interacting decision makers is large, the average decision does not fluctuate much away from the fixed point $E_\mu(X)$ in both cases. Social interactions and personal inertia then have similar effects, at least in terms of what can be seen in aggregate data.

As we have seen, social interactions and personal inertia lead to similar scenarios in terms of aggregate outcomes, making it difficult to identify the appropriate explanation from the observations. Without individual date, the level of variance of choices over time can be an indicator of whether reinforcement is generated by interactions or by inertia.

6 The influence of heterogeneity on the level of reinforcement:

In this section, we analyze how different properties of the characteristics distribution, in particular those related to its dispersion affect the level of reinforcement of 1-decisions. We provide some results in the setting of section 4, and more precisely under the assumptions of Proposition 3, when the long run distribution of decisions is given by the invariant measure μ. If, moreover, we adopt the assumptions of Theorem 5, the fraction of 1 choices is simply given by the fixed point of the choice probability which can easily be compared the expected fraction of 1-choices in a neutral environment providing us with a simple and precise measure of the magnitude of the reinforcement.

The literature on social interactions and in particular on binary choice suggests that greater heterogeneity of the characteristics distribution is likely to decrease the effects generated by social interactions. Weaker interactions give rise to weaker reinforcement effects in static equilibrium, e.g. Glaeser et al [18]. Greater heterogeneity of the characteristics distribution could be considered to weaken the strength of interactions, at least relatively, by making the private signal stronger, the extreme case being a distribution that places weight only on dominantly high and low types, so as to entirely preclude the effects of social interactions. Brock and Durlauf indeed find, in their model, that greater heterogeneity of the random taste variable reduces the effects of interactions (reduction of the multiplicity of equilibria) when comparing variables within the same class of distributions (extreme value distribution) but with different parameters and thus different variance. In the context of our model, we ask whether it holds true in general that greater heterogeneity translates into less reinforcement if we compare two arbitrary characteristics distributions that are ordered in terms of second order stochastic dominance.

In the asymptotic case, reinforcement is given by the difference between the fixed point and the average under ν which is $P(1|\frac{1}{2})$. We note that besides the value of $P(1|\frac{1}{2})$, the fixed point is not affected by the behavior of the probability on $[0, \frac{1}{2}]$. Indeed, if we modify $P(1|x)$ on $[0, \frac{1}{2}]$, without changing $P(1|\frac{1}{2})$, and in such a way that weak bias still holds, this has no effect on the fixed point but can modify the variance or the DS2 relation between measures. This observation about the choice probability translates into the following statement about characteristics:

Proposition 8 *Let Θ_1 and Θ_2 be two characteristics distributions which verify strong dominance of 1-types. If $P(\Theta_1 \leq s) = P(\Theta_2 \leq s)$ for all $s \in [-\infty, \theta_n]$, where θ_n is the neutral type, then the choice probabilities have the same fixed point and the reinforcement by interactions (or inertia) has the same magnitude in a large population.*

This proposition shows that within the class of distributions that verify strong dominance, the magnitude of the reinforcement depends on the distribution of low types but not on the exact distribution of high types. This was not apparent for distributions which are symmetric around the neutral type.

The previous result established, we analyze how the reinforcement depends on the distribution of Θ on $(-\infty, \theta_n]$. We find that a sufficient condition for increase not to occur is that P is flat on $[N/2, N]$ and that a sufficient condition for increase to occur is that P is increasing in the right neighborhood of $1/2$, or in terms of characteristics:

Proposition 9 *Under the assumption of Theorem 5, a sufficient condition for increase not to occur is that* $P(\theta \in]\theta_l, \theta_n[) = 0$.

Proposition 10 *Under the assumption of Theorem 5, a sufficient condition for increase to occur is that* $P(\theta \in [\theta_n - \epsilon, \theta_n[) > 0$.

Proposition 9 says that if all low types are dominantly low, then no increase is generated by social interactions. Proposition 10 states that positive increase will occur as long as there is some mass of types that are just slightly lower than neutral.

The conditions distinguish cases where there is a mass of types only far away from the neutral type or close to it. These conditions are thus indeed related to the homogeneity/heterogeneity of the distribution. However, it is easy to see that in general, the condition cannot be captured by second order stochastic dominance, the standard measure of the global dispersion ie heterogeneity of the distribution, nor of course by variance.

Proposition 11 *The magnitude of the reinforcement is not monotonous with respect to Stochastic second order dominance.*

Proof: we give a counter example. Let θ be such that $\theta_l < \theta_n < \theta < \theta_h$. Define $h(\theta)$ as the h such that $U(1, h, \theta) = U(0, h, \theta)$. Consider the following distribution of candidate characteristics: $P(\Theta) = \begin{cases} \theta_l & \frac{1}{6} \\ \theta & \frac{1}{2} \\ \theta_h & \frac{1}{3} \end{cases} \iff P(1|h) = \begin{cases} \frac{1}{3} & h \in [0, h(\theta)[\\ \frac{7}{12} & h = h(\theta) \\ \frac{5}{6} & h \in]h(\theta), 1] \end{cases}$

Since $\theta > \theta_n$, $h(\theta) < h(\theta_n) = \frac{1}{2}$. This distribution has $P(1|\frac{1}{2}) = \frac{5}{6} = x_f$. Therefore, the reinforcement is zero. We will now define $\tilde{\Theta}$ as a mean preserving spread of Θ. We redistribute the probability on θ symmetrically on $\theta_n < \theta$ and $\theta_m =: \theta + (\theta - \theta_n)$. We assume that $\theta - \theta_n$ is small so that $\theta_m =: \theta + (\theta - \theta_n) < \theta_h$. We have $0 < h(\theta_m) < h(\theta_n) = \frac{1}{2}$.

$$P(\tilde{\Theta}) = \begin{cases} \theta_l & \frac{1}{6} \\ \theta_n & \frac{1}{4} \\ \theta_m & \frac{1}{4} \\ \theta_h & \frac{1}{3} \end{cases} \iff P(1|h) = \begin{cases} \frac{1}{3} & h \in [0, h(\theta_m)[\\ \frac{11}{24} & h = h(\theta_m) \\ \frac{7}{12} & h \in]h(\theta_m), \frac{1}{2}] \\ \frac{17}{24} & h = \frac{1}{2} \\ \frac{5}{6} & h \in]\frac{1}{2}, 1] \end{cases}$$

. In absence of interactions, the expected frequency of 1 choices is $P(1|\frac{1}{2}) = \frac{11}{24}$, whereas the fixed point $x_f = \frac{5}{6} > \frac{11}{24}$. So in this case, the reinforcement is strictly positive.

We should note that while we gave an example where $P(1|x)$ is discontinuous, this discontinuity is not what drives the counter-example. We just took this case as it is simple to describe. We could have smoothed $P(1|x)$ and conserved a similar counter example.

This example shows that less heterogeneity in characteristics, as measured by second order stochastic dominance, does not in general ensure a stronger reinforcement effect due to social interaction and so the result of Brock and Durlauf with the logit distribution with different parameter cannot be generalized when we compare two arbitrary characteristics distributions.

Increasing characteristics heterogeneity using mean preserving spreads does not necessarily lead to weaker reinforcement. However, we can show that a different type of "spread" which instead of preserving the mean, preserves the proportion of 1-choices in absence of interactions, does have the property of reducing the reinforcement effect:

Proposition 12 *Let $\tilde{\Theta}$ be a spread of Θ that conserves the proportion of choices in a neutral environment and takes the following form:for a fixed $a > 1$, $\tilde{\Theta} = a\Theta + m(a)$, where $m(a) \in R$ is chosen to have $P_{\tilde{\Theta}}(1|\frac{1}{2}) = P_{\Theta}(1|\frac{1}{2})$. If $P_{\tilde{\Theta}}(1|h)$ and $P_{\Theta}(1|h)$ both have unique fixed points in $(\frac{1}{2}, 1]$, denoted $h_{\tilde{\Theta}}$ and h_{Θ} respectively, then $h_{\tilde{\Theta}} < h_{\Theta}$.*

Proof Let $a > 1$ be fixed and define $\tilde{\Theta} = a\Theta + m(a)$ where $m(a)$ is determined by the relation

$$P(U(1, a\Theta + m(a), \frac{1}{2}) > U(0, a\Theta + m(a), \frac{1}{2}) = P(U(1, \Theta, \frac{1}{2}) > U(0, \Theta, \frac{1}{2}). \tag{17}$$

We note that for every $h \in [0, 1]$ there is a $c(h)$, decreasing in h such that

$$P(1|h) = P(U(1, \Theta, h) > U(0, \Theta, h) = P(\Theta > c(h)). \tag{18}$$

Thus, (17) is equivalent to

$$P(a\Theta + m(a) > c(\frac{1}{2})) = P(\Theta > c(\frac{1}{2}). \tag{19}$$

Assuming that the cdf of Θ is strictly increasing, this implies $\frac{c(\frac{1}{2})-m(a)}{a} = c(\frac{1}{2})$.

For $h > \frac{1}{2}$, we have $c(h) < c(\frac{1}{2})$ and we can write $c(h) = c(\frac{1}{2}) - k$ with $k > 0$. Therefore we have

$$P_{\tilde{\Theta}}(1|h) = P(a\Theta + m(a) > c(h)) = P(a\Theta + m(a) > c(\frac{1}{2}) - k) = P(\Theta > \frac{c(\frac{1}{2}) - m(a)}{a} - \frac{k}{a}).$$

Since $\frac{c(\frac{1}{2})-m(a)}{a} = c(\frac{1}{2})$, $\frac{c(\frac{1}{2})-m(a)}{a} - \frac{k}{a} = c(\frac{1}{2}) - \frac{k}{a} > c(\frac{1}{2}) - k = c(h)$,

whence $P(\Theta > \frac{c(\frac{1}{2})-m(a)}{a} - \frac{k}{a}) < P(\Theta > c(h)) = P_{\Theta}(1|h)$, so finally $P_{\tilde{\Theta}}(1|h) < P_{\Theta}(1|h)$.

Now, suppose that $\bar{h}$ is the largest value of h such that $P_{\Theta}(1|\bar{h}) = \bar{h}$. By the assumption that $P_{\Theta}(1|\ 1) < 1$, we must have for every $h > \bar{h}$ that $P_{\Theta}(1|h) < h$. Since $P_{\tilde{\Theta}}(1|h) < P_{\Theta}(1|h)$ for every $h \geq \bar{h}$, necessarily the largest fixed point of $P_{\tilde{\Theta}}(1|h)$ is inferior to $\bar{h}$.

Proposition 12 can be applied for example to gaussian or uniform distributions, when restricting attention to distributions with the same proportion of 1 choices in a neutral environment.

We illustrate Proposition 12 by a numerical example. We use the utility function $U_i(a_i, h_i, \theta) = \beta(h_i - \frac{1}{2})(a_i - \frac{1}{2}) + (\theta - \frac{1}{2})(a_i - \frac{1}{2})$, and we fix $\beta = \frac{1}{2}$ and we consider gaussian distributions of the candidate characteristics. For every value of the variance σ, we fix the mean of the characteristics distribution in such a way that in absence of interaction or personal reinforcement, 60 percent of the candidates are above the neutral type and would be assigned decision 1. In absence of interactions, the frequency of 1 decisions would be the same in the populations whose characteristics the gaussian distributions with different variance. Table 1 shows how the fixed point varies as a function of σ. With low variance 1-choice frequencies could be modified to much higher values, increasing the frequency of decision 1 by almost 30 percent. When candidates are more heterogenous the level of reinforcement is much more modest.

Example 13 Table 1

variance	*m(σ)*	*fixed–point*	*reinforcement*
$\sigma = 1$	*0.76*	*0.62*	*0.02*
$\sigma = 0.9$	*0.73*	*0.63*	*0.03*
$\sigma = 0.8$	*0.71*	*0.64*	*0.04*
$\sigma = 0.7$	*0.68*	*0.65*	*0.05*
$\sigma = 0.6$	*0.66*	*0.67*	*0.06*
$\sigma = 0.5$	*0.60*	*0.67*	*0.07*
$\sigma = 0.4$	*0.58*	*0.69*	*0.09*
$\sigma = 0.3$	*0.55*	*0.74*	*0.14*
$\sigma = 0.2$	*0.53*	*0.88*	*0.28*

6.1 Skewed distributions- minorities with extreme characteristics

So far, we have focused on the case where there was a strong initial 1- bias. This assumption guaranteed further reinforcement of the decision towards which there was initial bias when the number of decision makers was large. Even under this strong condition, reinforcement was not ensured for a fixed finite number of decision makers. However, quantitatively, in the example we found of this, the reinforcement in the opposite direction was very small when strong dominance was verified. If we assume only a weak bias towards 1, in the sense $E[\Theta] > \theta_n$, that is expected type is greater than the neutral type, then a simple example shows that interactions can actually strongly reinforce the decision that is not suited to the majority. This occurs when there is a minority with characteristics highly biased to the opposite action. Let the utility function be $U_i(a_i, h_i, \theta) = \beta(h_i - \frac{1}{2})(a_i - \frac{1}{2}) + (\theta - \frac{1}{2})(a_i - \frac{1}{2})$, with $\beta = \frac{1}{2}$ for which the neutral type is $\frac{1}{2}$. We specify a distribution of characteristics such that $E[\Theta] > \frac{1}{2}$ is verified but $P(1|x) > P(0|1-x)$ is not. We let the distribution of characteristics be discrete and assume that there are only three types whose probabilities are :

$$\begin{cases} P(\Theta = 0) = \frac{9}{50} \\ P(\Theta = \frac{7}{10}) = \frac{4}{5} \\ P(\Theta = 1) = \frac{1}{50} \end{cases} \tag{20}$$

The ϵ are to avoid indifference of the decision maker. We note that while the expected value is greater than $\frac{1}{2}$, there are more extreme low than extreme high types. We fix $N = 2$, so that $h \in \{0, \frac{1}{2}, 1\}$. Our specification of utility is then such that if $h = 0$, the decision maker always chooses 1 only for types 1, if $h = \frac{1}{2}$, he chooses 1 for types $\theta > 1/2$ and if $h = 1$, he chooses 1 for all types except 0. In terms of choice probability,

$$\begin{cases} P(1|0) = \frac{1}{50} \\ P(1|\frac{1}{2}) = 41/50 \\ P(1|1) = \frac{41}{50} \end{cases} \tag{21}$$

Explicit computations of the invariant distribution gives

$$\begin{cases} \mu(0) = \frac{81}{140} \\ \mu(1) = \frac{9}{70} \\ \mu(2) = \frac{41}{140} \end{cases} \tag{22}$$

The average frequency of 1 choices under this distribution is $\frac{1}{N}\sum \mu(k)k = \frac{5}{14} < \frac{1}{2}$, which should be compared with 82 % 1-decisions in absence of interactions. Thus, interactions have increased the decision that is less suited to the majority. Models of social interactions often show a reinforcement of the action preferred by the majority, to a point where the minority suffers. This example, although anecdotal, indicates that social interactions can also under some conditions lead to reinforcement of a choice that is not suited to the majority.

7 Concluding discussion

Statistical discrimination is normally assumed to result from the decision maker's attempt to resolve a problem of incomplete information regarding the candidates type. This paper highlights two additional mechanisms, social influence between decision makers or personal inertia of the decision maker himself, whose effects are similar to those of statistical discrimination in that individuals' outcomes are adversely affected by belonging to a group with an unfavorable statistical distribution of characteristics.

Most of the results have already been discussed in the sections where they were presented. Let us summarize the main findings: we are able to characterize long run behavior of decisions and thus measure the reinforcement of action 1 (reject candidate), imputable to influence/inertia compared to the rate in a neutral environment in some particular cases. Under the condition of strong bias which expresses the fact that a group has unfavorable characteristics, we are able to show a reinforcement of rejection rates in some particular cases. We do not have an analytical result in general but our results span two opposite extremes in terms of assumptions : we can characterize the aggregate decision frequencies when utility and characteristics are chosen, essentially so as to make the choice probability linear in the influence variable. We showed in this case that the assumption of social influence and that of personal inertia or a combination of both generate positive reinforcement of similar magnitude. Secondly, in two special cases we obtain a complete characterization of the invariant distribution of decisions. One concerns uniform interactions and no personal inertia, the other pure personal inertia. These two cases are in fact similar from an analytical point of view and underscore again the fact that social interactions and personal inertia can both generate similar reinforcement effects at least in terms of the magnitude of the reinforcement.

As we have emphasized from the beginning, the hypotheses about available information that lead to our results are almost the opposite from the assumptions that lead to statistical discrimination in Phelps' seminal model. At a closer look, however, the processes at play have some similarities. In our case, also, the driving factor is the fact that the candidate belongs to a group with unfavorable characteristics. The decision maker does not know the statistical distribution but becomes aware of it through a form of social or individual learning, by observing the decisions of others and/or drawing on his own past experience. This process is successful in the sense that the decision maker does indeed "learn" the statistical distribution of the candidates group (or at least learns it to be unfavorable) which he initially ignored. However, the successful learning of the statistical properties of the group is in fact detrimental since the decision maker is assumed to observe the true type of the candidate. In this respect, we see something similar to irrational herding where agents' own informative signals are crowded out by social observations.

Turning now to the details of how properties of the characteristics distribution affect long run outcomes, it is necessary to distinguish the case with a finite fixed number of agents and asymptotic results with respect to population size. It is only asymptotically that the condition of strong bias ensures a systematic reinforcement of the rejection rate compared to the one seen in absence of interactions. In the finite case however, the strong bias condition is not sufficient to ensure an increase. We find cases where the long run distribution first order stochastically dominates respectively is dominated by the distribution in absence of interactions.

The findings of our study also confirms the important role of heterogeneity, which has been emphasized in the literature. However, the relevant notion that governs the magnitude of rejection increase is not captured by second order stochastic dominance. Instead we find a condition related to the probability of types close to the "neutral" one, a condition that is in a sense local, whereas the exact distribution of high types is not important. This was not clear when looking at a symmetric distribution as in the Brock and Durlauf model. Finally, examples show that for more complex distributions, for example, when groups are on average low skilled but also contain a highly skilled minority, the effects of social interactions are less predictable and can in fact lead to a lower rejection rate than what would otherwise be the case.

8 Appendix: Proofs

8.1 Proof of 2

We define recursive equations for the expectations of the aggregate state variables P^t and V^t. At each instant t, the agent i makes a choice a_i^t that depends on two state variables, one corresponding to the previous actions of his neighbors, v^{t-1} and the other one corresponding to his own previous decisions p^{t-1} These variables are defined recursively at time t as $v_i^{t+1} = \lambda_2 v_i^t + (1-\lambda_2)\frac{1}{CardV(i)}\sum_{j\in V(i)} a_j^t$ and $p_i^{t+1} = \lambda_1 p_i^t + (1-\lambda_1)a_i^{t+1}$ respectively. We note that since all agents have the same degree l,

$$\frac{1}{Card(V(i))}\sum_i \sum_{j\in V(i)} a_j^t = \sum_i a_i^t. \tag{23}$$

At a given time t, we can define the aggregate variable $V^t = (1/N)\sum_i v_i^t$ and $P^t = (1/N)\sum_i p_i^t$. We can now consider the conditional expectation, and conditionally on

$(v_i^t)_i$ and $(p_i^t)_i$, the variables $(a_i^{t+1})_i$ are independent. We have

$$
\begin{aligned}
&E[V^{t+1}|(v_i^t)_i,(p_i^t)_i] = \\
&\quad \lambda_2 V^t + (1-\lambda_2)\textstyle\sum_{i=1}^{i=N} \frac{1}{Card(V(i))} \sum_{j\in V(i)} P(a_j^{t+1}=1)|(v_i^t)_i,(p_i^t)_i) = \\
&\quad\quad \lambda_2 V^t + (1-\lambda_2)\textstyle\sum_{i=1}^{i=N}[(P_c-\frac{\beta}{2}) + \beta\gamma p_i^t + \beta(1-\gamma)v_i^t)] = \\
&\quad [\beta(1-\lambda_2)(1-\gamma)+\lambda_2)]V^t + [\beta(1-\lambda_2)\gamma]P^t + [1-\lambda_2](P_c-\tfrac{\beta}{2}).
\end{aligned} \tag{24}
$$

This last expression is (V^t, P^t) mesurable, and by properties of conditional expectation equals $E[V^{t+1}|V^t, P^t]$. Similarly we obtain

$$E[P^{t+1}|V^t,P^t] = [\beta(1-\lambda_1)\gamma]V^t + [\beta(1-\lambda_1)(1-\gamma)+\lambda_1)]P^t + [1-\lambda_1](P_c-\frac{\beta}{2})$$

Taking expectations on both sides of (24), we obtain a recursive relation:

$$E[V^{t+1}] = [\beta(1-\lambda_2)(1-\gamma)+\lambda_2)]E[V^t] + [\beta(1-\lambda_2)\gamma]E[P^t] + [1-\lambda_2](P_c-\frac{\beta}{2})$$

This relation holds for all t. We put $a = \beta(1-\lambda_2)(1-\gamma)+\lambda_2$, $b = \beta(1-\lambda_2)\gamma$, $c = \beta(1-\lambda_1)\gamma$ and $d = \beta(1-\lambda_1)(1-\gamma)+\lambda_1$. Then we obtain the difference equation

$$\begin{pmatrix} E[V^{t+1}] \\ E[P^{t+1}] \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} E[V^t] \\ E[P^t] \end{pmatrix} + (P_c - \frac{\beta}{2}) \begin{pmatrix} 1-\lambda_2 \\ 1-\lambda_1 \end{pmatrix}$$

We denote by

$$M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

We remark that a, b, c and d are all positive and satisfy $a+b<1$ and $c+d<1$ when $\beta<1$. Thus it is possible to write

$$M = \begin{pmatrix} a+b & 0 \\ 0 & c+d \end{pmatrix} T$$

where T is a stochastic matrix. We have

$$M^n = \begin{pmatrix} (a+b)^n & 0 \\ 0 & (c+d)^n \end{pmatrix} T^n$$

Since T is a stochastic matrix and $a+b<1$ and $c+d<1$, $\lim_{n\to\infty} M^n_{i,j} = 0$ for all i and j. Therefore, the solution of the homogenous equation associated with (25) goes to zero for n large. We can show that $v = p = \frac{P_c - \frac{1}{2}\beta}{1-\beta}$ is a particular solution to the inhomogenous equation since (v,p) satisfies the equations

$$p = [\beta(1-\lambda_1)\gamma]v + [\beta(1-\lambda_1)(1-\gamma)+\lambda_1)]p + [1-\lambda_1](P_c - \frac{1}{2}\beta)$$

$$v = [\beta(1-\lambda_2)(1-\gamma)+\lambda_2)]v + [\beta(1-\lambda_2)\gamma]p + [1-\lambda_2](P_c - \frac{1}{2}\beta).$$

Therefore, the general solution of the difference equation is such that

$$\lim_{t\to\infty} E[V^t] = \lim_{t\to\infty} E[P^t] = \frac{P_c - \frac{1}{2}\beta}{1-\beta} \tag{25}$$

8.2 proof of proposition 3 (average interaction case)

The invariant measure is unique. We use the notation a_k for any element $a \in \{0,1\}^N$ such that $\frac{1}{N}\sum_{i=1}^{i=N} = k$ and look for an invariant measure that puts the same weight on every element of type a_k. By the detailed balance conditions, an arbitrary fixed element represented by a_k can be reached from itself in two ways: by drawing one of the k agents who previously chose 1 when he chooses 1 again, or by drawing one of the $N-k$ agents who chose 0 before and whose new decision is still 0. An element a_k also has antecedents of type a_{k-1}. There are k such antecedents: any element where one of the k 1 choices in a_k is replaced by a 0 would be an antecedent. Similarly there are $N-k$ antecedents of type a_{k+1}. This gives the equation:

$$\mu(a_k) = \mu(a_k)\frac{k}{N}P(1|k) + \mu(a_k)\frac{N-k}{N}P(0|k) + \mu(a_{k-1})\frac{k}{N}P(1|k-1) + \\ \mu(a_{k+1})\frac{N-k}{N}P(0|k+1) \tag{26}$$

We will use the notation $\mu(k) =: \mu(\{a \in \{0,1\}^N | \frac{1}{N}\sum_{i=1}^{i=N} a_i = k\})$. Since $\mu(k) = C_k^N \mu(a_k)$, the relation (26)gives:

$$\mu(k) = \mu(k)\frac{k}{N}P(1|k) + \mu(k)\frac{N-k}{N}P(0|k) + \\ \mu(k-1)\frac{N-k+1}{N}P(1|k-1) + \mu(k+1)\frac{k+1}{N}P(0|k+1) \tag{27}$$

If we write these equations for $\mu(k)...\mu(0)$ and add them, we obtain:

$$\sum_{i=0}^{i=k}\mu(i) = \frac{1}{N}\sum_{i=0}^{i=k}\mu(i)[iP(1|i) + (N-i)P(1|i) + iP(0|i) + (N-i)P(0|i)] + \\ \frac{1}{N}\mu(k+1)(k+1)P(0|k+1) - \mu(k)\frac{N-k}{N}P(1|k)$$

After cancellation of most terms we obtain the relation $\mu(k)\frac{N-k}{N}P(1|k) = \mu(k+1)\frac{k+1}{N}P(0|k+1) \iff \mu(k) = \frac{(N-(k-1))}{k}\frac{P(1|k-1)}{P(0|k)}\mu(k-1)$.

Recursively we obtain

$$\mu(k) = C_k^N \prod_{i=0}^{i=k-1} \frac{P(1|\frac{i}{N})}{P(0|\frac{i+1}{N})}\mu(0) \tag{28}$$

8.3 proof proposition 4

By the definition of the invariant distribution μ, we have

$$\frac{\mu(k+1)}{\mu(k)} = \frac{C_k^N P(1|\frac{k}{N})}{C_{k+1}^N P(0|\frac{k+1}{N})} \tag{29}$$

We have

$$\frac{\mu(k+1)}{\mu(k)} > \frac{P(1|\frac{k}{N})N-k}{P(0|\frac{k}{N})k+1} = \frac{P(1|\frac{k}{N})}{P(0|\frac{k}{N})}(\frac{N-k}{k} - \frac{N-k}{k(k+1)}) \tag{30}$$

$$\frac{\mu(k+1)}{\mu(k)} < \frac{P(1|\frac{k+1}{N})N-k}{P(0|\frac{k+1}{N})k+1} = \frac{P(1|\frac{k+1}{N})}{P(0|\frac{k+1}{N})}(\frac{N-(k+1)}{k+1} + \frac{1}{k+1}). \tag{31}$$

We can now write:

$$g(\frac{k+1}{N}) - \frac{P(1|\frac{k+1}{N})}{P(0|\frac{k+1}{N})(k+1)} < \frac{\mu(k+1)}{\mu(k)} < g(\frac{k}{N}) + \frac{P(1|\frac{k}{N})(N-k)}{P(0|\frac{k}{N})k(k+1)} \tag{32}$$

with $g(x) = \frac{P(1|x)(1-x)}{(1-P(1|x))x}$. We note that if we fix $\delta > 0$, then for $\frac{k}{N} \in [\delta, 1]$, the terms $R(N) = \frac{P(1|\frac{k+1}{N})}{P(0|\frac{k+1}{N})(k+1)}$ and $\tilde{R}(N) = \frac{P(1|\frac{k}{N})(N-k)}{P(0|\frac{k}{N})k(k+1)}$ decrease to 0 as N increases since $k > N\delta$.

8.4 proof Theorem 5

Let x_f denote the unique fixed point of $P(1|x)$ in $[\frac{1}{2}, 1]$. We note that due to strong bias, $P(1|\frac{1}{2}) > \frac{1}{2}$ and $g(\frac{1}{2}) > 1$. Fix $\epsilon > 0$ and define $A = [x_f - \epsilon, x_f + \epsilon]$. We need to bound $\mu(l)$ for all $l \in A^c$. Let $x \in N$ be such that $\frac{x}{N} \in A$. (such an x exists if N is sufficiently large). We define $min_{y\in[\frac{1}{2},\frac{1}{2}+\frac{\epsilon}{2}]} g(y) =: m_1 > 1$. Since $P(1|x)$ and thus $g(x)$ has at most a finite number of discontinuities, we may assume ϵ small enough that g is continuous on $[\frac{1}{2}, \frac{1}{2} + \frac{\epsilon}{2}]$. Moreover, we have $min_{y\in[\frac{1}{2}+\frac{\epsilon}{2}, x_f-\frac{\epsilon}{2}]} g(y) \leq 1$. Let us consider $m \in N$ such that $\frac{m}{N} \in [\frac{1}{2}, x_f - \epsilon]$. We have

$$\frac{\mu(x)}{\mu(m)} = \frac{(N-m)P(1|\frac{m}{N})}{xP(0|\frac{x}{N})} \prod_{l=m+1}^{l=x-1} \frac{P(1|\frac{l}{N})}{P(0|\frac{l}{N})} \frac{N-l}{l} = \frac{(N-m)P(1|\frac{m}{N})}{xP(0|\frac{x}{N})} \prod_{l=m+1}^{l=x-1} g(\frac{l}{N})$$

For $\frac{m}{N} \in [\frac{1}{2}, x_f - \epsilon]$, the product contains at least $\frac{N\epsilon}{2}$ terms that are larger than m_1 and the other terms can be bounded below by 1. Thus $\mu(x) \geq (m_1)^{\frac{N\epsilon}{2}}\mu(m)$. Similar arguments show that for all $l \in [x_f + \epsilon, 1]$, there is $m_2 > 1$ such that $\mu(x) \geq (m_2)^{\frac{N\epsilon}{2}}\mu(l)$.

When $\frac{l}{N} \in [0, \frac{1}{2}]$ we have $\mu(l) = \mu(N-l)\prod_{i=l}^{i=N-l}\frac{P(1|\frac{i}{N})}{P(0|1-\frac{i}{N})}$. By hypothesis $\frac{P(1|x)}{P(0|1-x)} \geq 1$ for all $x \in [0,1]$. We can write $l = N-k$, where $\frac{k}{N} \in [\frac{1}{2}, 1]$. If $\frac{k}{N} \in [\frac{1}{2}, x_f - \epsilon]\bigcup[x_f + \epsilon, 1]$, then $\mu(l) \leq \mu(k)$, and we have already bounded $\mu(k)$. If $l = N-k$ with $k \in [x_f - \epsilon, x_f + \epsilon]$, then $\mu(l) = \mu(N-l)\prod_{i=l}^{i=N-l}\frac{P(1|\frac{i}{N})}{P(0|1-\frac{i}{N})}$. We have $x_f > \frac{1}{2}$ and we may assume $x_f - \frac{1}{2} \geq 3\epsilon$. For ϵ small enough, we can assume that $g(x)$ does not have discontinuities on $[x_f - 2\epsilon, x_f - \epsilon]$. Define $min_{x\in[x_f-2\epsilon, x_f-\epsilon]}\frac{P(1|x)}{P(0|1-x)} = m_3 > 1$. Thus the product contains at least ϵ terms, each one greater than m_3. Since there are less than N terms of the form $\frac{k}{N}$, $k \in N$ in $[0,1]$ and applying to each one the bounds we have established, finally, $\frac{\mu(A)}{\mu(A^c)} > \frac{1}{N}(min(m_1, m_2, m_3))^{\frac{N\epsilon}{2}}$. This quantity is greater than M for sufficiently large N.

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